RC Phase-Shift Oscillators

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Oscillators generate periodic signals in the time domain. They convert DC power into AC signal power. Signal generation implies production of self-sustained oscillations. According to the types of waveforms produced oscillators can be classified into one of four generic types:

-*Harmonic oscillators*: used for sine-wave generation.
-*Sawtooth oscillators*: used for the generation of exponential or linear sawtooth waves.
-*Relaxation oscillators*: used for current or voltage pulse generation with negative resistance devices.
-*Astable multivibrators*: used for the generation of rectangular or square waves.

Each type of oscillator resorts to either a positive-feedback or negative-resistance principle to operate. On the other hand, very complex waveforms can be generated mixing analog and digital techniques.

The Positive Feedback Approach for a Harmonic Oscillator
As a system using positive feedback, the harmonic oscillator can be represented in block-diagram form as depicted by Fig.1, where *G(s)* is the Laplace transform of the open-loop voltage-gain function of the amplifier stage and *H(s)* is the transfer function of the passive feedback network.

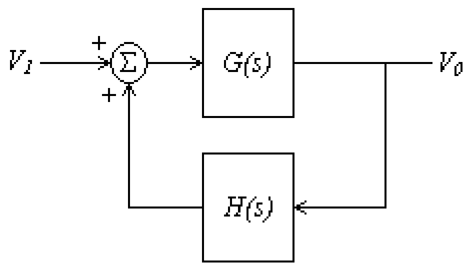


Fig.1 An oscillator as a feedback system

The closed-loop gain function of the system is found to be:

$$\frac{V_0}{V_1}(s) = \frac{G(s)}{1 - G(s)H(s)}$$

Mathematically, it represents the response of the system to an impulse function, i.e., a function for which $V_1(s) = 1$. In the real-world, impulse functions are approximated by narrow noise impulses.

The expression:

$$1 - G(s)H(s) = 0 \qquad \ldots (1)$$

is known as the *characteristic equation* and contains important information regarding the system's stability. Generally speaking, the roots of the equation are of the type $s = \alpha \pm j\omega$. When $\alpha = 0$ the system is marginally stable and $s = \pm j\omega$. If certain conditions are met, this situation will describe an oscillator in the steady state delivering a constant-amplitude sine wave of radian frequency ω. A real-world oscillator requires that $\alpha > 0$ for oscillations to start and build up. The power supply start-up impulse or circuit thermal noise will generally trigger the response. Then, some type of amplitude stabilization mechanism in the system will gradually reduce α and stabilize the oscillations. An amplitude-stabilized oscillator will usually exhibit good frequency stability too. One final note here is that once the circuit bursts into oscillation no external perturbation $v_1(t)$ is needed for sustained operation.

Equation (1) tells us that after amplitude stabilization:

$$G(j\omega) \cdot H(j\omega) = 1 \qquad \ldots (2)$$

This is Barkhausen's criterion for oscillations. The left hand member is a complex number and consists of a real and an imaginary part. Thus, we may write the equation:

$$\mathrm{Re}(\omega) + j\,\mathrm{Im}(\omega) = 1 + j0$$
$$\mathrm{Re}(\omega) = 1$$
$$\mathrm{Im}(\omega) = 0$$

$\mathrm{Im}(\omega)$ equated to zero will generally give the frequency of oscillation. $\mathrm{Re}(\omega)$ equated to unity will yield the conditions to be met for oscillations.

If we open the loop at the input of block *G(s)* in Fig.1 and inject a probe signal $V_T(s)$ at this point, the output from block *H(s)* would be:

$$V_L(s) = V_T(s) \cdot G(s)H(s)$$

We now define the gain function:

$$A_L(s) = \frac{V_L(s)}{V_T(s)} = G(s)H(s)$$

This is the *loop gain* of our system. Barkhausen's criterion then states that the oscillator's loop gain must be unity for oscillations to develop in the circuit.

Some Phase-Shift Oscillators

The most simple RC phase-shift oscillator configuration uses three buffered RC cells and a voltage amplifier with very high input impedance and very low output impedance. Fig.2 shows a typical schematic.

Because the RC cells won't load each other, the loop gain may be found to be:

$$A_L(s) = G(s)H(s) = A\cdot\left(\frac{sRC}{sRC+1}\right)^3$$

$$= A\cdot\left(\frac{s^3R^3C^3}{s^3R^3C^3+3s^2R^2C^2+3sRC+1}\right)$$

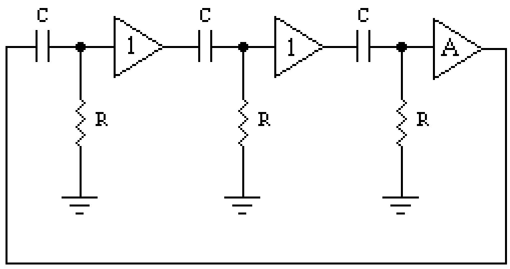


Fig.2 Simple RC Phase-Shift Oscillator

For sine-wave steady-state operation, $s = j\omega$, and the following may be written:

$$\frac{-j\omega^3R^3C^3}{\left(1-3\omega^2R^2C^2\right)+j\omega RC\left(3-\omega^2R^2C^2\right)} = \frac{1}{A} \qquad \ldots (3)$$

Equating the real part of the denominator to zero will yield the frequency of oscillation:

$$1-3\omega^2R^2C^2 = 0 \Rightarrow \omega_0 = \frac{1}{\sqrt{3}RC}$$

This result fed back into expression (3) will render information on amplifier gain A. Accordingly:

$$A = -8$$

The phase shift introduced by each RC cell can be obtained from its transfer function:

$$F(j\omega) = \frac{j\omega RC}{j\omega RC+1}$$

$$F(j\omega_0) = \frac{\dfrac{j}{\sqrt{3}}}{\left(\dfrac{j}{\sqrt{3}}+1\right)}$$

Then:

$$\phi = 90^{\circ} - \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = 90^{\circ} - 30^{\circ} = 60^{\circ}$$

The total phase shift introduced by the RC network at $\omega = \omega_0$ will be three times ϕ, or 180º.

The BJT RC Phase-Shift Oscillator

Alike the preceding configuration, the BJT RC Phase-Shift Oscillator is a popular configuration for the generation of low-frequency sine waves, starting at a few Hertz and up to about 100 kHz. A schematic diagram of a basic implementation can be seen in Fig.3. If the RC cells were isolated from each other, the phase shift per cell would be 60º. However, not being it the case, we need to perform a detailed analysis considering loading effects.

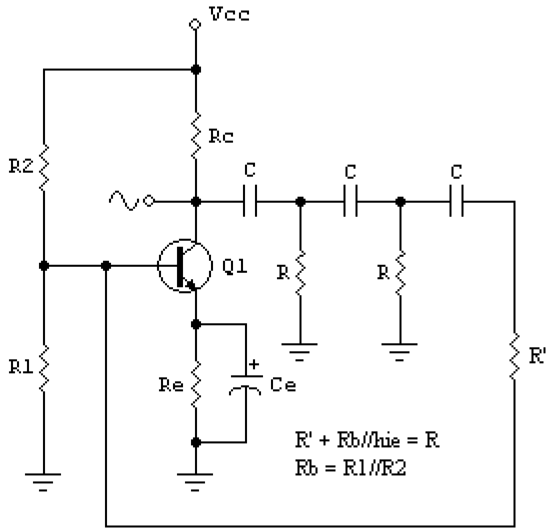


Fig.3 BJT-based RC Phase-Shift Oscillator

The bipolar transistor is a current amplifier. Therefore, it is convenient to express the loop gain as a current ratio:

$$A_L(s) = \frac{I_b}{I_b^{'}}$$

where $I_b^{'}$ is a test signal current injected at the transistor's base and I_b is the corresponding base current following loop excitation. Please see Fig.4.

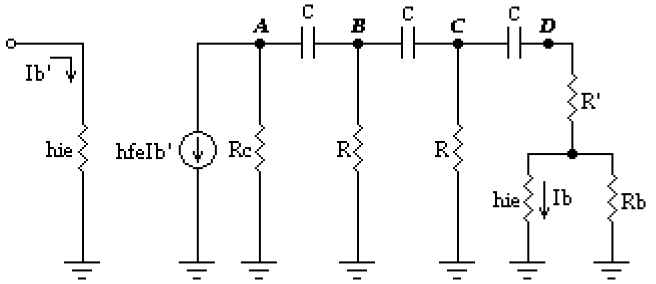


Fig.4 Equivalent circuit for loop-gain calculation

The node-potential set of equations in the Laplace domain can be written as:

$$-h_{fe}I_b' = \frac{V_A}{R_C} + (V_A - V_B)sC$$

$$0 = \frac{V_B}{R} + (V_B - V_A)sC + (V_B - V_C)sC$$

$$0 = \frac{V_C}{R} + (V_C - V_B)sC + (V_C - V_D)sC$$

$$0 = \frac{V_D}{R} + (V_D - V_C)sC$$

A small simplification leads to:

$$-h_{fe}I_b' = V_A\left(\frac{1}{R_C} + sC\right) - V_B \cdot sC \qquad ...(4.a)$$

$$0 = -V_A \cdot sC + V_B\left(\frac{1}{R} + 2sC\right) - V_C \cdot sC \qquad ...(4.b)$$

$$0 = -V_B \cdot sC + V_C\left(\frac{1}{R} + 2sC\right) - V_D \cdot sC \qquad ...(4.c)$$

$$0 = -V_C \cdot sC + V_D\left(\frac{1}{R} + sC\right) \qquad ...(4.d)$$

On the other hand, usually, $R_b >> h_{ie}$. Then:

$$V_D = RI_b \approx (R' + h_{ie})I_b \qquad ...(4.e)$$

From Eq.(4.d):

$$V_C = V_D\left(\frac{1}{sRC} + 1\right)$$

Substituting into Eq.(4.c):

$$0 = -V_B \cdot sC + V_D\left(\frac{1}{sRC}+1\right)\left(\frac{1}{R}+2sC\right) - V_D \cdot sC$$

which simplifies to:

$$V_B = V_D\left(\frac{1}{s^2R^2C^2}+\frac{3}{sRC}+1\right)$$

Substituting for V_B and V_C in Eq.(4.b) we get:

$$0 = -V_A sC + V_D\left(\frac{1}{s^2R^2C^2}+\frac{3}{sRC}+1\right)\left(\frac{1}{R}+2sC\right) - V_D\left(\frac{1}{sRC}+1\right)sC$$

yielding:

$$V_A = V_D\left(\frac{1}{s^3R^3C^3}+\frac{5}{s^2R^2C^2}+\frac{6}{sRC}+1\right)$$

Substituting for V_A and V_B in Eq.(4.a):

$$-h_{fe}I_b^{'} = V_D\left(\frac{1}{s^3R^3C^3}+\frac{5}{s^2R^2C^2}+\frac{6}{sRC}+1\right)\left(\frac{1}{R_C}+sC\right) - V_D\left(\frac{1}{s^2R^2C^2}+\frac{3}{sRC}+1\right)sC$$

After some simplification:

$$-h_{fe}I_b^{'} = V_D\left(\frac{1}{s^3R^3R_CC^3}+\frac{5}{s^2R^2R_CC^2}+\frac{6}{sRR_CC}+\frac{1}{R_C}+\frac{1}{s^2R^3C^2}+\frac{4}{sR^2C}+\frac{3}{R}\right)$$

Bearing in mind Eq.(4.e), the last expression transforms to:

$$-h_{fe}I_b^{'} = I_b\left(\frac{1}{s^3R^2R_CC^3}+\frac{5}{s^2RR_CC^2}+\frac{6}{sR_CC}+\frac{R}{R_C}+\frac{1}{s^2R^2C^2}+\frac{4}{sRC}+3\right) \qquad ...(5)$$

If $A_L(s)=1$, then $I_b = I_b^{'}$. For sine wave operation, $s = j\omega$ and Eq.(5) is rewritten as:

$$-h_{fe} = j\frac{1}{\omega^3R^2R_CC^3} - j\frac{6}{\omega R_CC} - j\frac{4}{\omega RC} - \frac{5}{\omega^2RR_CC^2} - \frac{1}{\omega^2R^2C^2} + \frac{R}{R_C} + 3 \qquad ...(6)$$

The frequency of oscillation is obtained equating the imaginary part of the right-hand member to zero:

$$0 = \frac{1}{\omega_0{}^3R^2R_CC^3} - \frac{6}{\omega_0R_CC} - \frac{4}{\omega_0RC}$$

Knowing that ω_0 can not be zero we may write:

$$0 = \frac{1}{\omega_0^{\,2} R^2 R_C C^2} - \frac{6}{R_C} - \frac{4}{R}$$

and solve for ω_0:

$$\omega_0 = \frac{1}{RC\sqrt{R_C\left(\frac{6}{R_C} + \frac{4}{R}\right)}}$$

or in a more suitable form:

$$\omega_0 = \frac{1}{RC\sqrt{6 + \frac{4R_C}{R}}}$$

in radians per second. The condition for oscillation is obtained equating to $-h_{fe}$ the real part of the right-hand member of Eq.(6) while making $\omega = \omega_0$:

$$\begin{aligned} -h_{fe} &= 3 + \frac{R}{R_C} - \frac{5}{\omega_0^{\,2} R R_C C^2} - \frac{1}{\omega_0^{\,2} R^2 C^2} \\ &= 3 + \frac{R}{R_C} - 5\left(\frac{6R}{R_C} + 4\right) - \left(6 + \frac{4R_C}{R}\right) \\ &= -23 - 29\frac{R}{R_C} - 4\frac{R_C}{R} \end{aligned}$$

Then:

$$h_{fe} = 23 + 29\frac{R}{R_C} + 4\frac{R_C}{R} \qquad \text{...(7)}$$

The minimum value of h_{fe} required for oscillations is $h_{fe\,\min} = 44.5$, and occurs when $\frac{R}{R_C} = 0.37$. If h_{fe} is less than the said value the circuit won't oscillate, because $A_L(j\omega)$ would be less than unity. We can write Eq.(7) in the alternate form:

$$\frac{R}{R_C} = \frac{h_{fe} - 23}{58} + \sqrt{\left(\frac{h_{fe} - 23}{58}\right)^2 - \frac{4}{29}}$$

Again, h_{fe} must be greater than 44.5 for $\frac{R}{R_C}$ to be a real number. If $h_{fe} = 44.5$, then $\frac{R}{R_C} = 0.37$. The design process would be then: Given h_{fe} and ω_0, find $\frac{R}{R_C}$. Then compute the RC product and select convenient values for C and R. Design the DC bias network for Class-A operation and symmetrical signal excursion. Amplitude distortion at the output may be reduced by introducing negative feedback in the emitter branch with a series added small resistor. Of course, a little more gain would be needed to compensate for the reduction in the effective transconductance $g_m^{'}$ of the circuit, i.e.:

$$g_m^{'} = \frac{g_m}{1 + g_m R_e}$$

where g_m is the transconductance of the transistor and R_e is the series added resistor.

The JFET RC Phase-Shift Oscillator

Another approach uses a discrete high input-impedance voltage amplifier stage. The JFET is a high input-impedance semiconductor device and is very well suited for voltage amplification. Fig.5 shows a schematic diagram of a typical JFET-based RC Phase-Shift oscillator, yet another popular alternative for low-frequency sine wave generation. As may be noted, resistor *R* of the far right RC cell substitutes for the gate-source bias return resistor commonly used in biasing schemes.

A general approach for the passive phase-shift network used in this type of oscillator can be seen in Fig.6, where Z_1 and Z_2 are a capacitor C and a resistor R. Either impedance can be selected to be the capacitor. However, the preferred configuration is that depicted by Fig.5. Given the JFET's high input impedance, loading effects occurring in the gate circuit are virtually eliminated. The amplifying device is biased for Class-A operation and minimum signal distortion at the output. If needed, negative feedback may be used for wave shape correction, as in the bipolar transistor case.

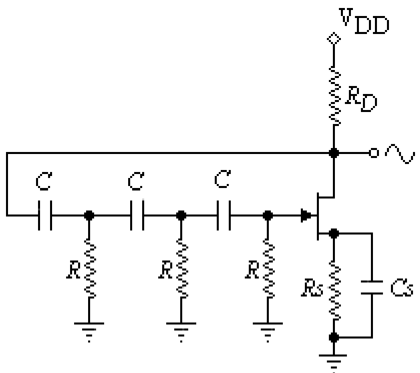


Fig.5 JFET-based RC Phase-Shift Oscillator

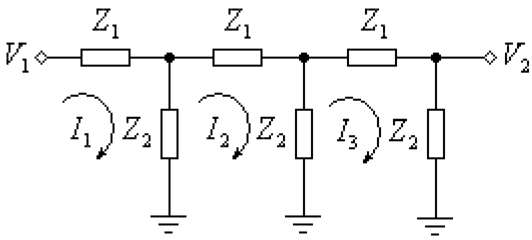


Fig.6 The three-cell RC network general approach

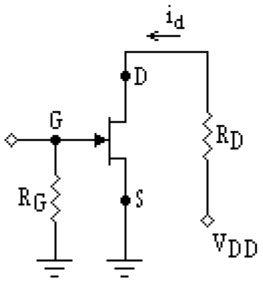
Design work requires some knowledge of the JFET's small-signal parameters. These are defined by:

$$r_{ds} = \frac{v_{ds}}{i_d} \quad when \ v_{gs} = 0$$

$$g_m = \frac{i_d}{v_{gs}} \quad when \ v_{ds} = 0$$

$$\mu = -\frac{v_{ds}}{v_{gs}} \quad when \ i_d = 0$$

where v_{ds}, i_d, v_{gs} are small-signal variations about a quiescent point (please see Fig.7 below).



$$i_d = g_m v_{gs} + \frac{v_{ds}}{r_{ds}}$$

$$v_{ds} = i_d r_{ds} - \mu v_{gs}$$

$$\mu = r_{ds} g_m$$

Fig.7 Basic JFET amplifier and small-signal equations

Here, g_m is the low-frequency forward transconductance, r_{ds} is the drain's dynamic output resistance and μ is the amplification factor. The latter equals the product $g_m r_{ds}$.

Sine wave operation permits us to perform calculations in the frequency domain. Accordingly, we can write the mesh equations for the network of Fig.6 in phasor form as:

$$V_1 = I_1(Z_1 + Z_2) - I_2 Z_2$$
$$0 = -I_1 Z_2 + I_2(Z_1 + 2Z_2) - I_3 Z_2$$
$$0 = -I_2 Z_2 + I_3(Z_1 + 2Z_2)$$

The output voltage is given by:

$$V_2 = I_3 Z_2 \qquad ...(8)$$

By Cramer's rule:

$$I_3 = \frac{\Delta_3}{\Delta}$$

The determinant of the *coefficient matrix* is calculated as:

$$\Delta = \begin{vmatrix} (Z_1 + Z_2) & -Z_2 & 0 \\ -Z_2 & (Z_1 + 2Z_2) & -Z_2 \\ 0 & -Z_2 & (Z_1 + 2Z_2) \end{vmatrix}$$

which reduces to:

$$\Delta = (Z_1 + Z_2)\begin{vmatrix} (Z_1 + 2Z_2) & -Z_2 \\ -Z_2 & (Z_1 + 2Z_2) \end{vmatrix} + Z_2[(-Z_2)(Z_1 + 2Z_2)]$$

After performing the indicated algebraic operations we obtain:

$$\Delta = Z_1^{\,3} + 5Z_1^{\,2}Z_2 + 6Z_1Z_2^{\,2} + Z_2^{\,3} \qquad ...(9)$$

The determinant Δ_3 is defined by:

$$\Delta_3 = \begin{vmatrix} (Z_1 + Z_2) & -Z_2 & V_1 \\ -Z_2 & (Z_1 + 2Z_2) & 0 \\ 0 & -Z_2 & 0 \end{vmatrix}$$

$$= V_1 Z_2^{\,2}$$

Then:

$$I_3 = \frac{Z_2^{\,2}}{Z_1^{\,3} + 5Z_1^{\,2}Z_2 + 6Z_1Z_2^{\,2} + Z_2^{\,3}} V_1$$

Substituting into Eq.(8):

$$V_2 = \frac{Z_2^{\,3}}{Z_1^{\,3} + 5Z_1^{\,2}Z_2 + 6Z_1Z_2^{\,2} + Z_2^{\,3}} V_1$$

The RC network's transfer function is then:

$$\frac{V_2}{V_1} = \frac{Z_2^{\,3}}{Z_1^{\,3} + 5Z_1^{\,2}Z_2 + 6Z_1Z_2^{\,2} + Z_2^{\,3}} \qquad ...(10)$$

The network's input impedance is given by:

$$Z_{IN} = \frac{V_1}{I_1} \qquad ...(11)$$

Observing that:

$$I_1 = \frac{\Delta_1}{\Delta} \qquad \text{...(12)}$$

where:

$$\Delta_1 = \begin{vmatrix} V_1 & -Z_2 & 0 \\ 0 & (Z_1 + 2Z_2) & -Z_2 \\ 0 & -Z_2 & (Z_1 + 2Z_2) \end{vmatrix}$$

$$= V_1\left[(Z_1 + 2Z_2)^2 - Z_2^{\ 2}\right]$$

$$= V_1\left(Z_1^{\ 2} + 4Z_1Z_2 + 3Z_2^{\ 2}\right) \qquad \text{...(13)}$$

we get, from equations (9), (11), (12) and (13):

$$Z_{IN} = \frac{V_1\Delta}{\Delta_1} = \frac{Z_1^{\ 3} + 5Z_1^{\ 2}Z_2 + 6Z_1Z_2^{\ 2} + Z_2^{\ 3}}{Z_1^{\ 2} + 4Z_1Z_2 + 3Z_2^{\ 2}} \qquad \text{...(14)}$$

Next, we choose Z_1 and Z_2 to be:

$$Z_1 = \frac{1}{sC}$$

$$Z_2 = R$$

Substituting in equations (10) and (14) we arrive to:

$$\frac{V_2}{V_1} = \frac{s^3R^3C^3}{s^3R^3C^3 + 6s^2R^2C^2 + 5sRC + 1} \qquad \text{...(15)}$$

and:

$$Z_{IN} = \frac{s^3R^3C^3 + 6s^2R^2C^2 + 5sRC + 1}{3s^3R^2C^3 + 4s^2RC^2 + sC}$$

$$= R\frac{s^3R^3C^3 + 6s^2R^2C^2 + 5sRC + 1}{3s^3R^3C^3 + 4s^2R^2C^2 + sRC} \qquad \text{...(16)}$$

From Eq.(15), and for $s = j\omega$:

$$H(j\omega) = \frac{V_2}{V_1}(j\omega) = -\frac{j\omega^3R^3C^3}{\left(1 - 6\omega^2R^2C^2\right) + j\omega RC\left(5 - \omega^2R^2C^2\right)}$$

From Eq.(16), and for $s = j\omega$:

$$Z_{IN} = R\frac{\left(1-6\omega^2R^2C^2\right)+j\omega RC\left(5-\omega^2R^2C^2\right)}{-4\omega^2R^2C^2+j\omega RC\left(1-3\omega^2R^2C^2\right)} \quad \text{...(17)}$$

With regards to Fig.5, the expression for the voltage gain of the amplifying device is:

$$A_V(j\omega) = -g_m(R_0 // Z_{IN})$$

where $R_0 = r_{ds} // R_D$, and Z_{IN} is the RC network's input impedance. Barkhausen's criterion for the oscillator's loop gain, Eq.(2), states that:

$$A_V(j\omega)\cdot H(j\omega) = -g_m(R_0 // Z_{IN})\cdot H(j\omega) = 1 + j0$$

where $H(j\omega) = \frac{V_2}{V_1}(j\omega)$ is the passive network's transfer function already calculated.

If we assume that the RC network doesn't load the JFET's output, a situation that can be met with an adequate selection of circuit values, Barkhausen's criterion will read as:

$$A_V(j\omega)\cdot H(j\omega) = -g_m R_0 \cdot H(j\omega) = 1 + j0$$

Substituting for $H(j\omega)$ in the loop-gain condition yields:

$$g_m R_0 \cdot \frac{j\omega^3R^3C^3}{\left(1-6\omega^2R^2C^2\right)+j\omega RC\left(5-\omega^2R^2C^2\right)} = 1 + j0$$

which is satisfied when:

$$1-6\omega^2R^2C^2 = 0$$

This sets the radian frequency of oscillation at:

$$\omega_0 = \frac{1}{\sqrt{6}RC}$$

The condition for oscillation is:

$$g_m R_0 \cdot \frac{\omega_0{}^2R^2C^2}{5-\omega_0{}^2R^2C^2} = 1$$

which shows that the minimum value required of voltage gain is $g_m R_0 = 29$. At the frequency of oscillation the input impedance of the RC network is given by Eq.(17) as:

$$\begin{aligned} Z_{IN} &= \frac{j29}{-4\sqrt{6}+j3}R \text{ ohms} \\ &= (0.83 - j2.70)R \text{ ohms} \\ &= 2.82R\exp(-j72.9^\circ) \text{ ohms} \end{aligned}$$

An interesting result is noticed here, and is that Z_{IN} is independent of the frequency of oscillation, depending only upon R.

In order to avoid loading sensibly the JFET's output, the following should be satisfied as a rule of thumb:

$$2.82R >> R_0 \Rightarrow R >> 0.355R_0 \Rightarrow R > 3.55R_0$$

We would now like to consider the effect of the RC network loading the JFET's output. Observing Fig.4 we can write the following circuit analogies:

$$V_{gs}^{'} \Leftrightarrow V_{be}^{'}$$

$$V_{gs} \Leftrightarrow V_{be} = I_b R$$

for the case in which $R^{'} = 0$. Then:

$$\frac{V_{gs}}{g_{m\,JFET} V_{gs}^{'}} \Leftrightarrow \frac{V_{be}}{g_{m\,BJT} V_{be}^{'}} \Leftrightarrow \frac{I_b R}{h_{fe} I_b^{'}}$$

Barkhausen's criterion stablishes that:

$$\frac{V_{gs}}{V_{gs}^{'}} \Leftrightarrow \frac{I_b}{I_b^{'}} = 1$$

We may write then:

$$\frac{1}{g_{m\,JFET}} \Leftrightarrow \frac{R}{h_{fe}}$$

and:

$$h_{fe} \Leftrightarrow g_{m\,JFET} R \qquad \text{...(18)}$$

We already arrived, in the BJT case to:

$$h_{fe} > 23 + 4k + \frac{29}{k} \quad with\ k = \frac{R_0}{R}$$

Using analogy (18):

$$g_{m\,JFET} R > 23 + 4k + \frac{29}{k}$$

$$g_{m\,JFET} \cdot \frac{R_0}{k} > 23 + 4k + \frac{29}{k}$$

$$g_{m\,JFET} R_0 > 23k + 4k^2 + 29$$

Finally, the minimum voltage gain required from the JFET stage considering loading effects is:

$$A_v = g_m R_0 = 4\left(\frac{R_0}{R}\right)^2 + 23\left(\frac{R_0}{R}\right) + 29$$

where g_m, R_0 and R are quantities previously defined during the analysis process of the JFET oscillator.

The Five-Cell RC Phase-Shift Oscillator

The five-cell RC phase-shift oscillator is an extension of the three-cell case of the preceding section, and will also require a high input-impedance voltage amplifier for ease of analysis and design. A medium input-impedance amplifier could also be used, but Z_2 would need to be considered part of the amplifier's input circuitry, requiring further analysis prior to design work. There exists also some concern with regards to the loading effect of the RC network on the amplifier's output. This issue will be given detailed consideration in our study. As will be demonstrated, the use of five RC cells reduces considerably the gain threshold for oscillation, as compared to the three-cell circuit. The amplifying device will be considered to be a vacuum triode. Fig.8 shows a five-cell RC network as used in a phase-shift oscillator.

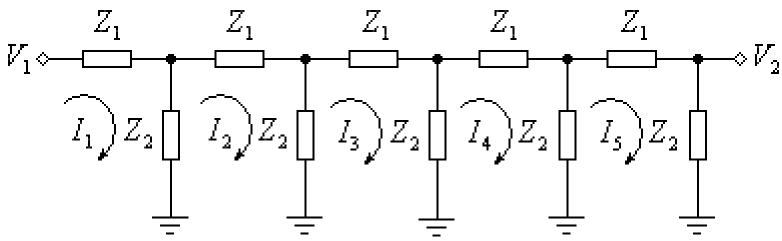


Fig.8 Five-cell RC network

The mesh equations for the passive network with input V_1 may be written in phasor form as:

$$V_1 = I_1(Z_1 + Z_2) - I_2 Z_2$$
$$0 = -I_1 Z_2 + I_2(Z_1 + 2Z_2) - I_3 Z_2$$
$$0 = -I_2 Z_2 + I_3(Z_1 + 2Z_2) - I_4 Z_2$$
$$0 = -I_3 Z_2 + I_4(Z1 + 2Z_2) - I_5 Z_2$$
$$0 = -I_4 Z_2 + I_5(Z_1 + 2Z_2)$$

and $V_2 = I_5 Z_2$. The system of five equations with five unknowns will be solved using, again, Cramer's rule. The determinant of the *coefficient matrix* is:

$$\Delta = \begin{vmatrix} (Z_1+Z_2) & -Z_2 & 0 & 0 & 0 \\ -Z_2 & (Z_1+2Z_2) & -Z_2 & 0 & 0 \\ 0 & -Z_2 & (Z_1+2Z_2) & -Z_2 & 0 \\ 0 & 0 & -Z_2 & (Z_1+2Z_2) & -Z_2 \\ 0 & 0 & 0 & -Z_2 & (Z_1+2Z_2) \end{vmatrix}$$

which can be reduced to:

$$\Delta = (Z_1+Z_2)\begin{vmatrix} (Z_1+2Z_2) & -Z_2 & 0 & 0 \\ -Z_2 & (Z_1+2Z_2) & -Z_2 & 0 \\ 0 & -Z_2 & (Z_1+2Z_2) & -Z_2 \\ 0 & 0 & -Z_2 & (Z_1+2Z_2) \end{vmatrix} - Z_2^2\begin{vmatrix} (Z_1+2Z_2) & -Z_2 & 0 \\ -Z_2 & (Z_1+2Z_2) & -Z_2 \\ 0 & -Z_2 & (Z_1+2Z_2) \end{vmatrix}$$

After calculating the determinants and performing the remaining algebraic operations we arrive to:

$$\Delta = Z_1^5 + 9Z_1^4 Z_2 + 28Z_1^3 Z_2^2 + 35Z_1^2 Z_2^3 + 15Z_1 Z_2^4 + Z_2^5$$

Mesh current I_5 is calculated from $I_5 = \frac{\Delta_5}{\Delta}$. The determinant Δ_5 is given by:

$$\Delta_5 = \begin{vmatrix} (Z_1+Z_2) & -Z_2 & 0 & 0 & V_1 \\ -Z_2 & (Z_1+2Z_2) & -Z_2 & 0 & 0 \\ 0 & -Z_2 & (Z_1+2Z_2) & -Z_2 & 0 \\ 0 & 0 & -Z_2 & (Z_1+2Z_2) & 0 \\ 0 & 0 & 0 & -Z_2 & 0 \end{vmatrix}$$

which readily simplifies to:

$$\begin{aligned} \Delta_5 &= Z_2\begin{vmatrix} (Z_1+Z_2) & -Z_2 & 0 & V_1 \\ -Z_2 & (Z_1+2Z_2) & -Z_2 & 0 \\ 0 & -Z_2 & (Z_1+2Z_2) & 0 \\ 0 & 0 & -Z_2 & 0 \end{vmatrix} \\ &= Z_2^2\begin{vmatrix} (Z_1+Z_2) & -Z_2 & V_1 \\ -Z_2 & (Z_1+2Z_2) & 0 \\ 0 & -Z_2 & 0 \end{vmatrix} \\ &= Z_2^3\begin{vmatrix} (Z_1+Z_2) & V_1 \\ -Z_2 & 0 \end{vmatrix} \\ &= V_1 Z_2^4 \end{aligned}$$

Then:

$$V_2 = I_5 Z_2 = \frac{\Delta_5}{\Delta} Z_2 = \frac{Z_2^5}{\Delta} V_1$$

The RC network's transfer function is defined as $H(j\omega) = \frac{V_2}{V_1} = \frac{Z_2^5}{\Delta}$, yielding:

$$H(j\omega)=\frac{Z_2^{\,5}}{Z_1^{\,5}+9Z_1^{\,4}Z_2+28Z_1^{\,3}Z_2^{\,2}+35Z_1^{\,2}Z_2^{\,3}+15Z_1Z_2^{\,4}+Z_2^{\,5}}$$

Next, we need to compute the RC network's input impedance. It is given by:

$$Z_{IN}=\frac{V_1}{I_1}$$

with current I_1 obtained from $I_1=\dfrac{\Delta_1}{\Delta}$.

$$\Delta_1=\begin{vmatrix} V_1 & -Z_2 & 0 & 0 & 0 \\ 0 & (Z_1+2Z_2) & -Z_2 & 0 & 0 \\ 0 & -Z_2 & (Z_1+2Z_2) & -Z_2 & 0 \\ 0 & 0 & -Z_2 & (Z_1+2Z_2) & -Z_2 \\ 0 & 0 & 0 & -Z_2 & (Z_1+2Z_2) \end{vmatrix}$$

which can be shown to reduce to:

$$\Delta_1=V_1(Z_1+2Z_2)\begin{vmatrix} (Z_1+2Z_2) & -Z_2 & 0 \\ -Z_2 & (Z_1+2Z_2) & -Z_2 \\ 0 & -Z_2 & (Z_1+2Z_2) \end{vmatrix}-V_1Z_2^{\,2}\begin{vmatrix} -(Z_1+2Z_2) & Z_2 \\ Z_2 & -(Z_1+2Z_2) \end{vmatrix}$$

It further simplifies to:

$$\Delta_1=-V_1(Z_1+2Z_2)\left(-Z_1^{\,3}-6Z_1^{\,2}Z_2-10Z_1Z_2^{\,2}-4Z_2^{\,3}\right)-V_1\left(Z_1^{\,2}Z_2^{\,2}+4Z_1Z_2^{\,3}+3Z_2^{\,4}\right)$$

giving:

$$\Delta_1=V_1\left(Z_1^{\,4}+8Z_1^{\,3}Z_2+21Z_1^{\,2}Z_2^{\,2}+20Z_1Z_2^{\,3}+5Z_2^{\,4}\right)$$

Then:

$$Z_{IN}=\frac{V_1}{I_1}=\frac{V_1\Delta}{\Delta_1}=\frac{\Delta}{\left(\frac{\Delta_1}{V_1}\right)}=\frac{Z_1^{\,5}+9Z_1^{\,4}Z_2+28Z_1^{\,3}Z_2^{\,2}+35Z_1^{\,2}Z_2^{\,3}+15Z_1Z_2^{\,4}+Z_2^{\,5}}{Z_1^{\,4}+8Z_1^{\,3}Z_2+21Z_1^{\,2}Z_2^{\,2}+20Z_1Z_2^{\,3}+5Z_2^{\,4}}$$

When Z_1 is a capacitor *C* and Z_2 a resistor *R*, the impedances are $Z_1=\dfrac{1}{j\omega C}$ and $Z_2=R$. Hence, substituting for Z_1 and Z_2 we obtain:

$$Z_{IN} = \frac{-j\dfrac{1}{\omega^5 C^5} + \dfrac{9R}{\omega^4 C^4} + j\dfrac{28R^2}{\omega^3 C^3} - 35\dfrac{R^3}{\omega^2 C^2} - j\dfrac{15R^4}{\omega C} + R^5}{\dfrac{1}{\omega^4 C^4} + j\dfrac{8R}{\omega^3 C^3} - 21\dfrac{R^2}{\omega^2 C^2} - j\dfrac{20R^3}{\omega C} + 5R^4}$$

Multiplying the numerator and denominator of the above expression by $\omega^5 RC^5$ we get:

$$Z_{IN} = R\frac{\omega^5 R^5 C^5 - j15\omega^4 R^4 C^4 - 35\omega^3 R^3 C^3 + j28\omega^2 R^2 C^2 + 9\omega RC - j}{5\omega^5 R^5 C^5 - j20\omega^4 R^4 C^4 - 21\omega^3 R^3 C^3 + j8\omega^2 R^2 C^2 + \omega RC}$$

Rearranging real and imaginary terms:

$$Z_{IN} = R\frac{\omega^5 R^5 C^5 - 35\omega^3 R^3 C^3 + 9\omega RC + j\left(-15\omega^4 R^4 C^4 + 28\omega^2 R^2 C^2 - 1\right)}{5\omega^5 R^5 C^5 - 21\omega^3 R^3 C^3 + \omega RC + j\left(-20\omega^4 R^4 C^4 + 8\omega^2 R^2 C^2\right)} \quad \ldots(19)$$

Considering the loading effect of the RC network upon the triode's output yields the familiar expression for the voltage gain of the vacuum tube:

$$A_V(j\omega) = -g_m(R_0 // Z_{IN})$$

where $R_0 = r_P // R_P$. The two paralleled resistances here are the plate's dynamic output resistance and the external plate bias resistor, respectively. Barkhausen's criterion for the oscillator's loop gain, Eq.(2), gives:

$$A_V(j\omega)\cdot H(j\omega) = -g_m(r_P // R_P // Z_{IN})\cdot H(j\omega) = 1 + j0$$

where $H(j\omega) = \frac{V_2}{V_1}(j\omega)$ is the passive network's transfer function. Substituting for $A_V(j\omega)$ and $H(j\omega)$ their individual expressions we obtain for the loop gain:

$$A_V(j\omega)\cdot H(j\omega) = -g_m(R_0 // Z_{IN})\cdot\frac{Z_2^{\,5}}{Z_1^{\,5} + 9Z_1^{\,4}Z_2 + 28Z_1^{\,3}Z_2^{\,2} + 35Z_1^{\,2}Z_2^{\,3} + 15Z_1Z_2^{\,4} + Z_2^{\,5}}$$

$$= \frac{-g_m\left(\dfrac{R_0 Z_{IN}}{R_0 + Z_{IN}}\right)Z_2^{\,5}}{Z_1^{\,5} + 9Z_1^{\,4}Z_2 + 28Z_1^{\,3}Z_2^{\,2} + 35Z_1^{\,2}Z_2^{\,3} + 15Z_1Z_2^{\,4} + Z_2^{\,5}}$$

$$= \frac{-gm\left(\dfrac{R_0 Z_{IN}}{R_0 + Z_{IN}}\right)Z_2^{\,5}}{Z_{IN}\left(Z_1^{\,4} + 8Z_1^{\,3}Z_2 + 21Z_1^{\,2}Z_2^{\,2} + 20Z_1Z_2^{\,3} + 5Z_2^{\,4}\right)}$$

After some simple algebraic work and simplification we arrive to:

$$A_V(j\omega)\cdot H(j\omega)=\frac{-g_m R_0 Z_2^5}{R_0\left(Z_1^4+8Z_1^3Z_2+21Z_1^2Z_2^2+20Z_1Z_2^3+5Z_2^4\right)+\left(Z_1^5+9Z_1^4Z_2+28Z_1^3Z_2^2+35Z_1^2Z_2^3+15Z_1Z_2^4+Z_2^5\right)}$$

Substituting for Z_1 and Z_2 their respective expressions, we get, after some manipulation:

$$A_V(j\omega)\cdot H(j\omega)=\frac{-g_m R_0 R^5}{\left[\frac{(R_0+9R)}{\omega^4C^4}-\frac{(21R_0R^2+35R^3)}{\omega^2C^2}+(5R_0R^4+R^5)\right]+j\left[-\frac{1}{\omega^5C^5}+\frac{(8R_0R+28R^2)}{\omega^3C^3}-\frac{(20R_0R^3+15R^4)}{\omega C}\right]}$$

$$=1+j0$$

The denominator of the fraction above must be a real negative quantity for the loop gain equation to hold. Then:

$$-\frac{1}{\omega^5C^5}+\frac{(8R_0R+28R^2)}{\omega^3C^3}-\frac{(20R_0R^3+15R^4)}{\omega C}=0$$

Knowing that ωC can not be equal to zero, the above equation simplifies to:

$$-\frac{1}{\omega^4C^4}+\frac{(8R_0R+28R^2)}{\omega^2C^2}-(20R_0R^3+15R^4)=0$$

which may be rewritten as:

$$(20R_0R^3+15R^4)\omega^4C^4-(8R_0R+28R^2)\omega^2C^2+1=0$$

or in the alternate form:

$$\left(20\frac{R_0}{R}+15\right)R^4\omega^4C^4-\left(8\frac{R_0}{R}+28\right)R^2\omega^2C^2+1=0$$

Solving for ω^2 we get:

$$\omega^2=\frac{\left(8\frac{R_0}{R}+28\right)\pm\sqrt{64\left(\frac{R_0}{R}\right)^2+368\left(\frac{R_0}{R}\right)+724}}{\left(40\frac{R_0}{R}+30\right)R^2C^2}=\frac{K^2}{R^2C^2} \qquad \ldots(20)$$

This equation gives the radian frequency of oscillation. We keep the positive sign for the square root above because, as can be shown, the negative sign would not permit satisfaction of the loop-gain condition.

We now need to calculate the minimum gain for oscillations to occur. The loop-gain equation dictates that:

$$\frac{-g_m R_0 R^5}{\frac{(R_0+9R)}{\omega^4 C^4} - \frac{(21R_0R^2+35R^3)}{\omega^2 C^2} + (5R_0R^4+R^5)} = 1$$

This expression can be rewritten as:

$$\frac{-g_m R_0}{\frac{\left(\frac{R_0}{R}+9\right)}{\omega^4 R^4 C^4} - \frac{\left(21\frac{R_0}{R}+35\right)}{\omega^2 R^2 C^2} + \left(5\frac{R_0}{R}+1\right)} = 1$$

and solved for $g_m R_0$, giving:

$$g_m R_0 = -\frac{\left(\frac{R_0}{R}+9\right)}{\omega^4 R^4 C^4} + \frac{\left(21\frac{R_0}{R}+35\right)}{\omega^2 R^2 C^2} - \left(5\frac{R_0}{R}+1\right) \quad \ldots(21)$$

This is the minimum value of the product $g_m R_0$ for sustained oscillations in the circuit. Tables I and II below show how the ratio $\frac{R_0}{R}$ influences results for the frequency of oscillation $fosc = \frac{K}{2\pi RC}$ and the triode's unloaded-case small-signal voltage gain, $g_m R_0$. The particular case $\frac{R_0}{R} \to 0$ corresponds to situations where the RC network won't load the vacuum tube's output.

TABLE I

$\frac{R_0}{R}$	$8\frac{R_0}{R}+28$	$40\frac{R_0}{R}+30$	$\left[64\left(\frac{R_0}{R}\right)^2+368\left(\frac{R_0}{R}\right)+724\right]^{\frac{1}{2}}$	$K^2=\omega^2R^2C^2$	$K=\omega RC$
1/2	32	50	30.39	1.248	1.117
1/4	30	40	28.63	1.466	1.211
1/8	29	35	27.77	1.622	1.273
1/16	28.5	32.5	27.33	1.718	1.311
1/20	28.4	32	27.25	1.739	1.319
1/40	28.2	31	27.08	1.783	1.335
0	28	30	26.91	1.830	1.353

TABLE II

$\frac{R_0}{R}$	$\frac{R_0}{R}+9$	$21\frac{R_0}{R}+35$	$5\frac{R_0}{R}+1$	$\omega^2R^2C^2$	$\omega^4R^4C^4$	g_mR_0
1/2	9.5	45.5	3.5	1.248	1.557	26.85
1/4	9.25	40.25	2.25	1.466	2.149	20.90
1/8	9.125	37.63	1.625	1.622	2.631	18.11
1/16	9.0625	36.31	1.313	1.718	2.951	16.75
1/20	9.05	36.05	1.25	1.739	3.024	16.49
1/40	9.025	35.53	1.125	1.783	3.179	15.96
0	9	35	1	1.830	3.349	15.44

We shall work out an example with the preceding results in an unusual application of a vintage 1629 magic-eye tube in a single-stage 1-Hz sine-wave oscillator. This circuit can be used to check the functionality of the tube while observing the opening and closing of the “eye” once per second. The 1629 is a later version of the original 6E5 magic-eye tuning indicator tube in an octal base and a 12.6V/0.15A filament-voltage regimen. It was manufactured by RCA, Tung-Sol and other companies. Characteristic curves of the 1629 are similar to those of the 6E5.

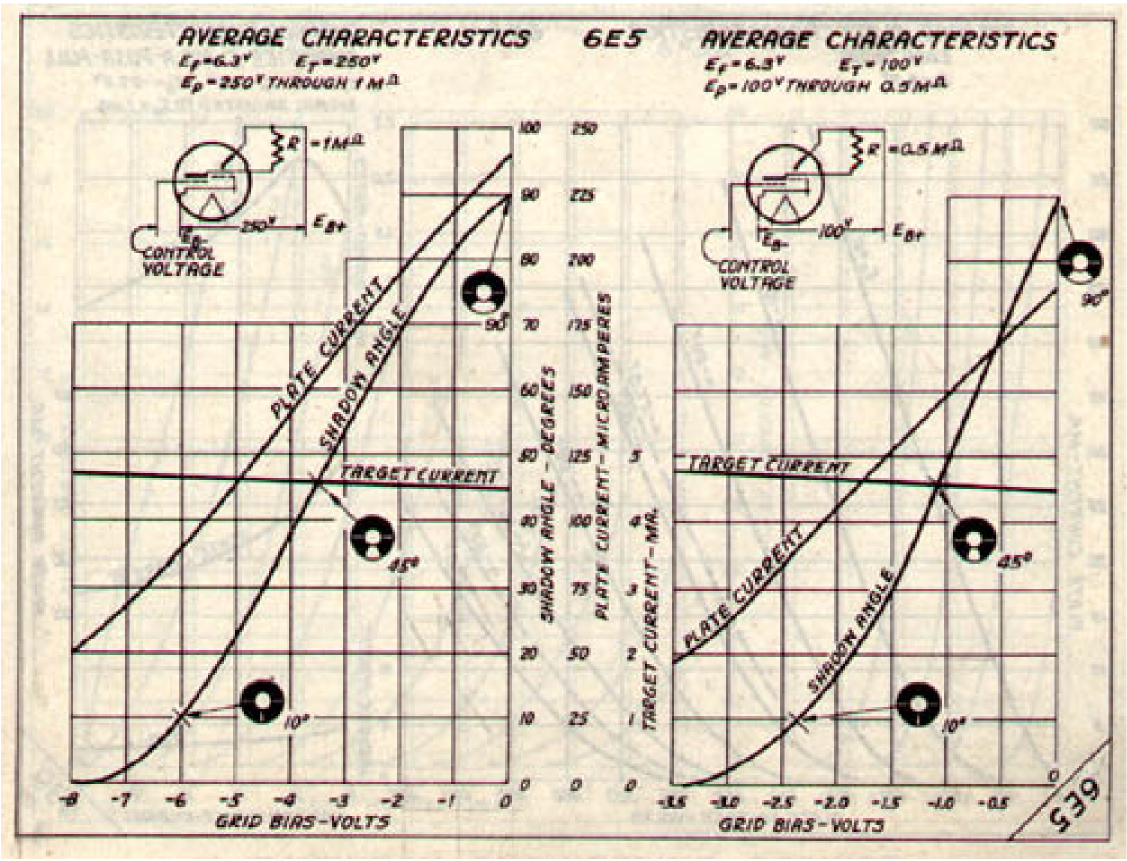
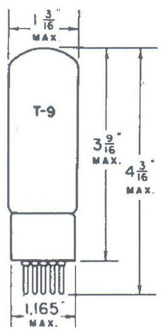


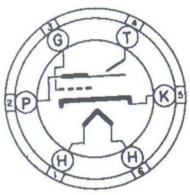
Fig.9 Average Characteristics of the 6E5 Magic-Eye tube

1629 (6E5)

TUNG-SOL



6E5



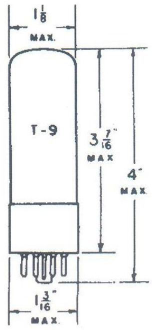
CATHODE RAY TUNING INDICATOR

UNIPOTENTIAL CATHODE

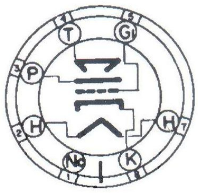
HEATERS

6E5 - 6.3 V., 0.3 AMPERE

1629 - 12.6 V., .15 AMPERE



1629



THE 1629 AND 6E5 ARE IDENTICAL WITH THE EXCEPTION OF THE HEATER RATINGS. THIS TUBE IS A CATHODE RAY TUNING INDICATOR WITH A CIRCULAR FLOURESCENT TARGET. THE DEGREE OF SHADOW ANGLE IS VARIED BY THE AVC OR OTHER CONTROLLING GRID VOLTAGE.

RATINGS

MAXIMUM PLATE SUPPLY VOLTAGE	250	VOLTS
MAXIMUM TARGET VOLTAGE	250	VOLTS
MINIMUM TARGET VOLTAGE	100	VOLTS

TYPICAL OPERATING CONDITIONS AND CHARACTERISTICS

PLATE AND TARGET SUPPLY VOLTAGE	100	200	250	VOLTS
TRIODE GRID VOLTAGE (0° SHADOW ANGLE)	-3.3	-6.5	-8.0	VOLTS
TRIODE GRID VOLTAGE (90° SHADOW ANGLE)	0	0	0	VOLTS
SERIES TRIODE PLATE RESISTOR	0.5	1	1	MEGOHM
TRIODE PLATE CURRENT (TRIODE GRID V.=0)	0.19	0.19	0.24	MA.
TARGET CURRENT (TRIODE GRID V.=0) ⁄	1	3	4	MA.

⁄ SUBJECT TO WIDE VARIATIONS

FOR "INTERPRETATION OF RATINGS" REFER TO FRONT OF BOOK

PLATE 1384-1
DEC. 15 1943

Fig.10 Data for the 1629 Magic-Eye tube

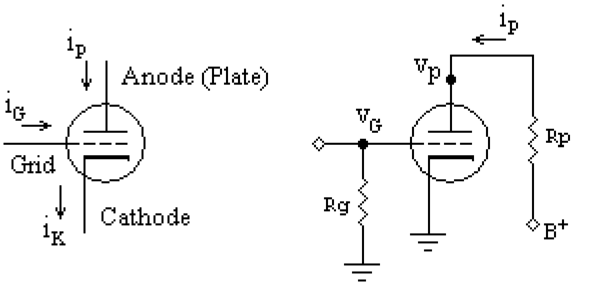
The oscillator makes use of the triode section of the 1629 magic-eye tube for signal amplification. The triode's plate potential to ground controls the opening and closing of the eye, following the 1-Hz oscillation. Design work requires some knowledge on the tube's triode small-signal parameters. For a graphical computation of these we can use the following definitions in conjunction with data obtained from the tube's characteristic curves:

$$r_p = \frac{v_p}{i_p} \; when \; v_{gk} = 0$$

$$g_m = \frac{i_p}{v_{gk}} \; when \; v_p = 0$$

$$\mu = -\frac{v_p}{v_{gk}} \; when \; i_p = 0$$

where v_p, i_p, v_{gk} are small-signal variations about a quiescent point (see Fig.11 below).



$$i_p = g_m v_{gk} + \frac{v_p}{r_p}$$

$$v_p = i_p r_p - \mu v_{gk}$$

$$\mu = r_p g_m$$

Fig.11 Basic triode amplifier and small-signal equations

The Tung-Sol 1629/6E5 specifications sheet shows data for DC power supply voltages of 100, 200 and 250 Volts. We have at our disposal a 150-Volt DC power supply on the workbench, so we will have to work out average 1629 data for this supply. The triode section's peak current will have to be limited to some 0.2 mA. This suggests a series triode plate resistor of 750k ohms. The triode grid voltage for a 0º shadow angle will be something in between -3.3 Volts and -6.5 Volts (cathode at zero potential). Choosing a geometric mean for this grid voltage, we get -4.63 Volts. One half of this value, or -2.32 Volts, will be the quiescent triode grid voltage for the best expected grid to cathode waveform. The small-signal plate output resistance r_p will be calculated upon this value.

The plate will be loaded externally by the 750k-ohm resistor paralleled by the RC network's input impedance Z_{IN}. We should not expect, then, a large net load impedance on the tube's triode plate. It would be advisable to maximize the plate's signal current excursion in this sense. For this reason, we select 0.1mA for the value of the quiescent triode plate current (about one half of the average peak plate current), assuming Class-A operation. On the graphs, for a plate current of 125uA, the average grid bias is -4.7 Volts with a 250-Volt supply and -1.5 Volts for a 100-Volt supply. The geometric mean is -2.65 Volts, close enough to our previously selected grid bias. In order to minimize estimation errors from the curves, we shall select a grid bias voltage of -2 Volts on the graphics. During implementation, this will require the use of an AC-decoupled cathode bias resistor.

We are now able to graphically (and grossly) estimate the small-signal parameters. For a 100-Volt DC supply and grid bias voltage of -2 Volts we find on the respective curve a plate current of 106.25 uA, which will be labeled as I_1. The external plate resistor is 0.5M ohms. The plate-to-cathode voltage drop is then V_{P1} = 100-(106.25)(0.5) = 46.87 Volts. On the other hand, for a 250-Volt DC supply and grid bias voltage of -2 Volts the plate current is I_2 = 193.75 uA. The external plate resistor is 1M ohm. Accordingly, the plate-to-cathode voltage drop is V_{P2} = 250-(193.75)(1) = 56.25 Volts. The small-signal plate output resistance r_p will be estimated from the expression:

$$r_p = \frac{V_{P2} - V_{P1}}{I_2 - I_1}$$

with grid bias voltage, V_{GK}, constant at -2 Volts. We get:

$$r_p = \frac{(56.25 - 46.87)Volts}{(193.75 - 106.25)\mu A} = \frac{9.38\ V}{87.5\ \mu A} = 107.2k\ ohms$$

We shall now proceed to estimate a value for the amplification factor μ. The calculation requires that the plate current stays constant at a certain fixed value, for instance, 125 μA (a value close to the selected 0.1-mA quiescent point current).

From the characteristic curves, for a 100-Volt DC supply, the plate current is 125 μA when the grid bias voltage (cathode at zero potential) is V_{GK1} = -1.5 Volts. The plate-to-cathode voltage drop is V_{P1} = 100-(125)(0.5) = 37.5 Volts. For a 250-Volt DC supply, the plate current is 125 μA when the grid bias voltage is V_{GK2} = -4.7 Volts. The plate-to-cathode voltage drop is V_{P2} = 250-(125)(1) = 125 Volts. The amplification factor μ is estimated from the expression:

$$\mu = \frac{V_{P2} - V_{P1}}{V_{GK1} - V_{GK2}}$$

with constant plate current equal to 125 μA. We get:

$$\mu = \frac{125 - 37.5}{-1.5 + 4.7} = 27.34$$

In terms of small-signal parameters, the amplification factor is $\mu = r_p g_m$. We can readily estimate the triode's transconductance as $g_m = \dfrac{\mu}{r_p} = 0.255\, mA/V$.

R_0 as defined before is given by:

$$R_0 = r_p \,//\, R_P = 107.2k\, ohms \,//\, 750k\, ohms = 93.79k\, ohms$$

The expression for the radian frequency of oscillation, Eq.(20), yields $\omega_{OSC} RC = 1.353$ when $\dfrac{R_0}{R} \to 0$. In this case, the RC network input impedance is found to be:

$$\begin{aligned} Z_{IN} &= R\frac{(69.953 + j0.005)}{(-27.993 - j52.34)}\; ohms \\ &= (0.5555 - j1.0388)R\; ohms \\ &= 1.178R\, \exp(-j61.865^{o})\, ohms \end{aligned}$$

after Eq.(19) and values from Tables I and II. Thus, for a given vacuum triode and bias resistors, Z_{IN} is independent of the frequency of oscillation, depending only upon the value selected for *R*.

As a rule of thumb for design, the RC network will not load the triode's plate output if the following is accomplished:

$$|Z_{IN}| >> R_0 \Rightarrow 1.178R >> R_0 \Rightarrow R >> 0.8489R_0 \Rightarrow R \geq 8.5R_0$$

At this point, we may start deciding on part values for the oscillator circuit. First, we find the ratio $\dfrac{R_0}{R}$, knowing that recommended values for R lie between 1M ohm and 3M ohms for the selected vacuum tube (triode section). A 1M-ohm value is chosen because it is a handy one. Then, $\dfrac{R_0}{R} = 0.09379$ and, from formulas previously developed, $\omega_{OSC} RC = 1.292$. For a 1-Hz oscillation we would need, accordingly, a 0.205-µF capacitor. A standard 0.22-µF part could be used if accuracy is not an issue.

To evaluate the loading effect of the passive network we just recall the Barkhausen criterion stating that:

$$-g_m (R_0 \,//\, Z_{IN}) \cdot H(j\omega) = 1 + j0$$

The RC network input impedance may be found to be:

$$
\begin{aligned}
Z_{IN} &= \frac{-60.177 + j3.974}{-25.9775 - j42.296}\, M\, ohms \\
&= (0.5663 - j1.0749)\, M\, ohms \\
&= 1.215 \exp(-j62.22^{o})\, M\, ohms
\end{aligned}
$$

The net plate load impedance is:

$$
\begin{aligned}
R_0 // Z_{IN} &= 93.79k\, ohms // (0.5663 - j1.0749)\, M\, ohms \\
&= \frac{(0.09379)\cdot(0.5663 - j1.0749)}{(0.09379 + 0.5663 - j1.0749)}\, M\, ohms \\
&= \frac{0.0531 - j0.1008}{(0.6601 - j1.0749)}\, M\, ohms \\
&= (90.094 - j5.943)\, k\, ohms \\
&= 90.29 \exp(-j3.77^{o})\, k\, ohms
\end{aligned}
$$

The overall voltage gain of the triode amplifier is:

$$
\begin{aligned}
A_V &= -g_m (R_0 // Z_{IN}) \\
&= -0.255\, mA/V \cdot [90.29 \exp(-j3.77^{o})]\, k\, ohms \\
&= [0.255 \exp(j180^{o})]\, mA/V . [90.29 \exp(-j3.77^{o})]\, k\, ohms \\
&= 23.02 \exp(j176.23^{o})
\end{aligned}
\quad \ldots (22)
$$

We see that the input impedance of the RC network introduces a 3.77° phase lag in the overall voltage gain, compared to the case where the plate load is purely resistive. According to Eq.(21), the triode's unloaded-case small-signal voltage gain, $g_m R_0$, requires to be greater than 17.42 for oscillations to start in the circuit. From experimental data, we are actually having:

$$
g_m R_0 = (0.255\, mA/V) \cdot (93.79\, k\, ohms) = 23.92
$$

enough gain for sustained oscillations to occur.

The successful 1-Hz oscillator

With theoretical and experimental data clearly understood, a working oscillator was built accordingly on a vintage KEL CORP. Model 5710-100-044 experimental printed circuit board (Fig.12). The 1629 magic-eye tube was plugged into a glass-ceramic 8-pin octal socket located on top of the circuit board. The RC network consisted of five 0.22uF/400V capacitors and five 1M-ohm 1/4-watt resistors. A 750k-ohm 1/4-watt resistor was used for plate bias from a 150-Volt DC power supply. A 5k-ohm potentiometer wired as a rheostat was connected in series with the cathode of the vacuum tube for adjustment of the optimal grid bias voltage. A 220V/12V 60-Hz 25-watt AC mains transformer heated the 12.6V/0.15A filament of the tube. The oscillation frequency, when checked with an electronic wrist watch, was found to be very close to the design value of 1Hz. As expected, the eye opened and fully closed once per second. The schematic diagram of the oscillator is shown in Fig.14.

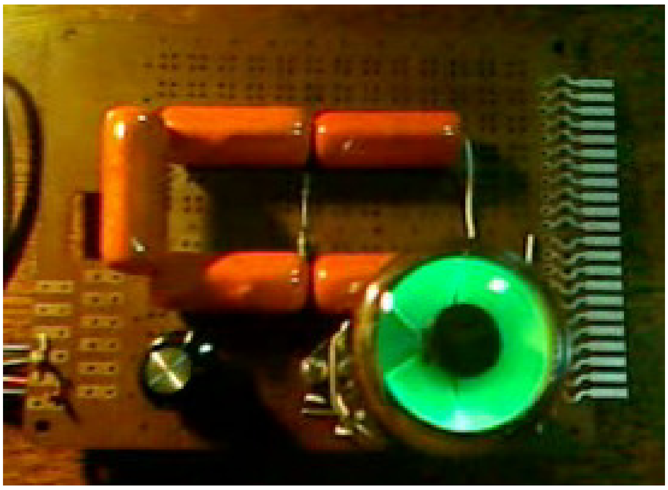
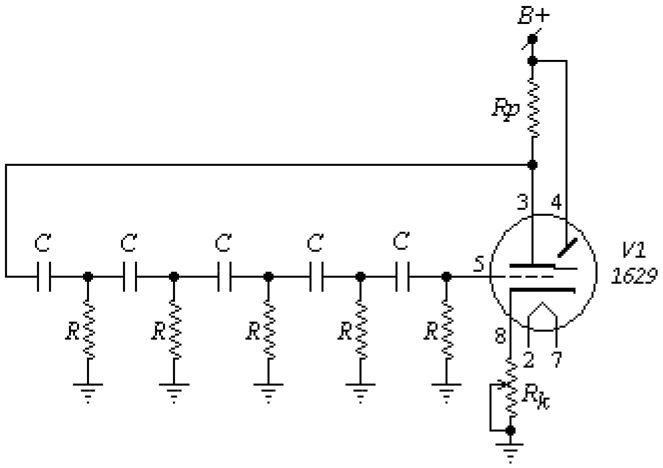


Fig.12 Top view of the 1-Hz sine-wave oscillator construction based on the 1629 Magic-Eye tube

Fig.13 View showing adjustment of grid voltage for maximum opening/closing delta-angle of the eye



B+ = 150VDC
Heater Pins 2 and 7 connect to a 12.6V-0.15A AC/DC supply
Rp = 750k ohms 1/4W
R = 1M ohm 1/4W
C = 0.22uF/400VDC
R_k is a 5k-ohm pot wired as rheostat

Notes: Adjust R_k for maximum eye closeness during oscillation. Cathode current is mostly the target's current. For the 150-Volt DC supply it varies between 0.6mA and 0.7mA when the eye is fully opened or closed. Cathode's potential approx. constant. No need for C_k. Oscillation frequency is 1Hz.

RC Phase-Shift Oscillator using a 1629 Magic-Eye Vacuum Tube as the active device
Fosc = 1Hz

Fig.14 Schematic diagram of the sine-wave 1-Hz oscillator

Measured data for the 1629 Magic-Eye tube used in the experimental oscillator

A. *Static bias operation measured with a B&K Test Bench 388A DMM*

-The plate current of the triode amplifier section reads 0.16mA when the cathode bias resistor R_K is set to zero ohms (cathode at ground potential, i.e., V_K = 0 Volts). Under these conditions, the triode will also exhibit a contact potential of -0.64 Volts, i.e., V_{GK} = -0.64 Volts.

-If V_K is increased to 1.2 Volts, the contact potential vanishes (V_G shows zero Volts).

-In order to inhibit oscillations in the circuit for static bias operation, one of the R-labeled resistors is shunted to ground with a jumper (do not select the 1M-ohm grid resistor). Under these conditions, an approximately 1º shadow angle is obtained with a cathode potential of V_K = 2.83 Volts. An 80º shadow angle is obtained with V_K = 0 Volts.

-At the operating point, the voltage drop across the 750k-ohm plate resistor is 103.7 Volts, yielding a plate current of 0.138 mA. Please compare with Eq.(23)

B. *Dynamic operation*

-Removing the jumper restores free oscillations and the eye opens and closes once per second. A totally closed eye followed by the widest opening occurs when V_K is adjusted to a quiescent value of 1.85 Volts. At this point, R_K measures 2.92k ohms, giving 0.633 mA for the total cathode current at the operating point. Although the potentiometer is a 5k-ohm part, out of the circuit shows a total resistance of 4.44k ohms, yielding 0.637 mA for the total cathode current at a 1º shadow angle.

-Our DMM has a peak-hold mode capability. Very careful measurements of the slowly varying DC voltages were taken in order to determine the peak signal values existing in the circuit. During oscillation, the voltage drop across the 750k-ohm plate resistor varies from 79.70 Volts to 128.20 Volts. Assuming Class-A operation, the mean value is:

$$\bar{V}_{Rp} = \frac{128.20 + 79.70}{2} = 103.95\,Volts \qquad \ldots(23)$$

The plate signal voltage is:

$$v_p = \frac{128.20 - 79.70}{2} = 24.25\,Volts$$

after observing that the cathode's potential is approximately constant during oscillation, due to the almost constant cathode current occurring during the opening and closing of the eye. The cathode doesn't need a decoupling capacitor in this application then.

The grid potential varies between -1.13 Volts and +0.76 Volts. The mean value is:

$$\bar{V}_g = \frac{0.76 - 1.13}{2} = -0.185\,Volts$$

The negative value is presumably due to contact potential action in the tube. If contact potential were zero, then $\bar{V}_g$ would be also zero for Class-A operation. The grid signal voltage is:

$$v_g = \frac{0.76 + 1.13}{2} = 0.945\,Volts$$

The grid-to-cathode voltage is defined as $V_{gk} = V_g - V_k$, where V_g and V_k are the DC plus AC voltages present at the grid and cathode electrodes, respectively. Hence, V_{gk} varies between (-1.13-1.85) Volts = -2.98 Volts and (0.76-1.85) Volts = -1.09 Volts. The most negative value, i.e., -2.98 Volts, is very close to the static value of -2.83 Volts needed for a 1º shadow angle. For a 0º shadow angle a slightly more negative voltage is needed, hence, the -2.98-Volt figure is reasonably correct. On the other hand, the mean value for V_{gk} is:

$$\bar{V}_{gk} = \frac{-1.09 - 2.98}{2} = -2.035\,Volts$$

The grid-to-cathode signal voltage is:

$$v_{gk} = \frac{-1.09 + 2.98}{2} = 0.945\,Volts = v_g$$

The magnitude of the overall voltage gain would be then:

$$A_v = \frac{v_p}{v_{gk}} = \frac{24.25}{0.945} = 25.66$$

reasonably close to the magnitude of the complex value given by Eq.(22).

We shall now refer to Fig.15. It shows the small-signal model for the triode section's output.

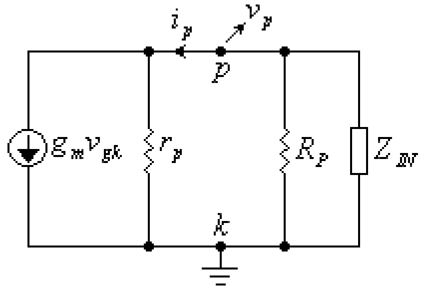


Fig.15 Triode section output equivalent

The following may be written:

$$-g_m v_{gk} = v_p \left(\frac{1}{r_p} + \frac{1}{R_P} + \frac{1}{Z_{IN}} \right)$$

An equivalent form of the above equation is:

$$g_m v_{gk} + \frac{v_p}{r_p} = i_p = -v_p \left(\frac{1}{R_P} + \frac{1}{Z_{IN}} \right)$$

The right hand member sets the contribution of external components R_P and Z_{IN}. Hereinafter, v_{gk}, v_p, i_p and Z_{IN} will be treated as phasors. Accordingly, if a 0º phase reference is assigned to v_p, then:

$$v_p = 24.25 \exp(j0^{\circ})\,Volts$$

The external plate load consists of $R_P = 750\,k\,ohms$ and $Z_{IN} = (0.5663 - j1.0749)M\,ohms$ connected in parallel:

$$\begin{aligned} R_P // Z_{IN} &= \frac{(0.75)\cdot(0.5663 - j1.0749)}{(0.75)+(0.5663 - j1.0749)} M\,ohms \\ &= \frac{0.4247 - j0.8062}{1.3163 - j1.0749} M\,ohms \\ &= \frac{0.9112\exp(-j62.22^{o})}{1.6994\exp(-j39.23^{o})} M\,ohms \\ &= 0.5362\exp(-j22.99^{o}) M\,ohms \end{aligned}$$

The plate signal current is given by:

$$\begin{aligned} i_p &= -\frac{v_p}{R_P // Z_{IN}} \\ &= -\frac{24.25\,Volts}{0.5362\exp(-j22.99^{o}) M\,ohms} \\ &= -45.22\exp(j22.99^{o}) uA \\ &= 45.22\exp(j202.99^{o}) uA \end{aligned}$$

and leads the plate signal voltage by 202.99º, or equivalently, lags the signal voltage by 157.01º.

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October 17th 2013