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Design of a Regenerative Receiver for the Short-Wave Bands
A Tutorial and Design Guide for Experimental Work

Part I

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Regenerative Receivers remain as the most popular choice among radio experimenters for Medium-Wave Broadcast-Band (MW BCB) and Short-Wave (SW) listening, due to their high performance, low complexity and ease of design. When correctly built they offer superb performance and less spurious products than consumer-grade superheterodyne counterparts.

The principle behind their operation is very simple: An RF oscillator with a loop gain slightly under unity can be made to work as a high-gain amplifier for radio signals, and when conveniently biased it will also recover AM modulation. An antenna system couples energy from passing radio waves to the oscillator's frequency-selective LC network.

The circuit needs some positive feedback or negative-resistance mechanism in order to achieve a high degree of amplification, and in this sense, virtually any RF oscillator configuration can be used. Amplification depends upon loop gain, and the amount of signal magnification is manually adjusted by the operator. There are a number of loop-gain control methods that the constructor can choose from, the selection being normally dictated by his personal preferences or the overall availability of components and parts. Known methods include adjustments on the bias of the detector-amplifier stage, the amount of positive-feedback or the tuning tank's damping factor.

For SSB decoding, the circuit is put into mild oscillation, operation requiring good frequency stability. The latter is accomplished designing the detector-amplifier for low DC power dissipation and using high-quality frequency-determining components. A stabilized power supply may be also recommended.

Positive feedback compensates for circuital RF resistive losses, being most critical those of the tuning network. Losses are indicative of RF power dissipation and they are expressed in terms of the tuning circuit's overall Q factor. Coils exhibit losses due to the conductor's resistivity and skin effect, and proximity effect between adjacent turns. Capacitors show dielectric losses, and those of the air-dielectric variable types, losses due to poor mechanical/electrical contact between the rotor and the capacitor's frame and also resistive and skin-effect losses on the stator and rotor plates.

The present report has been divided into two sections. Part I will review tuned-circuit design formulas and Part II will apply them in the calculation of a simple SW regenerative receiver covering the frequency band from 3MHz to 12MHz.

Part I
Review of Basic Tuned-Circuit Analysis

The Parallel-Tuned Circuit with Ideal Reactances

Consider the network of Fig.1. The inductor L and capacitor C form a parallel-tuned circuit loaded by resistor R. A signal current source *I(s)* in the Laplace domain drives the network, developing a voltage V(s) across the resistor and the two reactances.

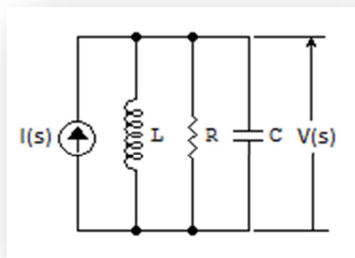


Fig.1 Parallel-tuned circuit with ideal reactances

It can be readily found that:

$$\frac{I(s)}{V(s)} = \frac{1}{sL} + sC + \frac{1}{R}$$

The impedance across the signal source is:

$$Z(s) = \frac{V(s)}{I(s)} = \frac{sL}{s^2LC + s\frac{L}{R} + 1}$$

For steady-state sine-wave operation, $s = j\omega$. Then:

$$Z(j\omega) = \frac{j\omega L}{(1-\omega^2 LC) + j\omega\frac{L}{R}}$$

$$= \frac{R}{1 + j\frac{R}{\omega L}(\omega^2 LC - 1)}$$

$$= \frac{R}{1 + jQ_T\frac{\omega_0}{\omega}\left(\frac{\omega^2}{\omega_0^2} - 1\right)}$$

where:

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

and

$$Q_T = \frac{R}{\omega_0 L}$$

The magnitude of *Z(jω)* is at a maximum when:

$$\frac{\omega^2}{\omega_0^2} - 1 = 0$$

i.e., when $\omega = \omega_0 = 2\pi f_0 = 1/\sqrt{LC}$. This is the resonant frequency of the network. Q_T is the associated overall Q factor, and it will govern selectivity and the -3dB bandwidth of the tuned circuit.

The above calculation yields:

$$|Z(j\omega_0)| = R$$

The magnitude of the impedance of the tuned circuit decreases above and below resonance, according to the function:

$$Z(\omega) = |Z(j\omega)| = \frac{R}{\sqrt{1 + Q_T^2 \frac{\omega_0^2}{\omega^2}\left(\frac{\omega^2}{\omega_0^2} - 1\right)^2}}$$

shown in Fig.2 as a normalized graph, first for $Q_T = 2$, and then for $Q_T = 5$. Plots show how Q_T affects selectivity, the ability of the tuned circuit for rejecting frequencies off-band.

The -3dB bandwidth is obtained equating *Z(ω)* to $R/\sqrt{2}$, i.e., making:

$$Q_T \frac{\omega_0}{\omega}\left(\frac{\omega^2}{\omega_0^2} - 1\right) = \pm 1$$

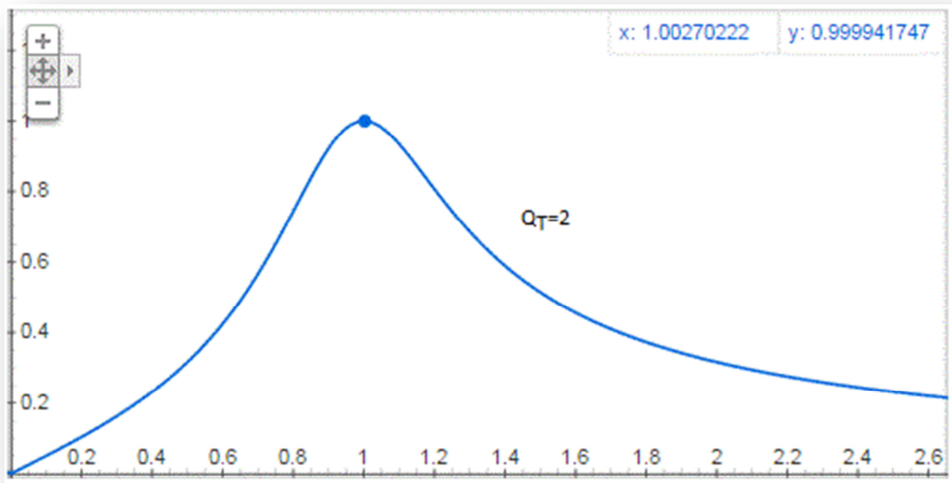
and solving the equation

$$\frac{\omega^2}{\omega_0^2} - 1 = \pm \frac{\omega}{Q_T \omega_0}$$

or equivalently

$$\frac{\omega^2}{\omega_0^2} \mp \frac{\omega}{Q_T \omega_0} - 1 = 0$$

for ω/ω_o.



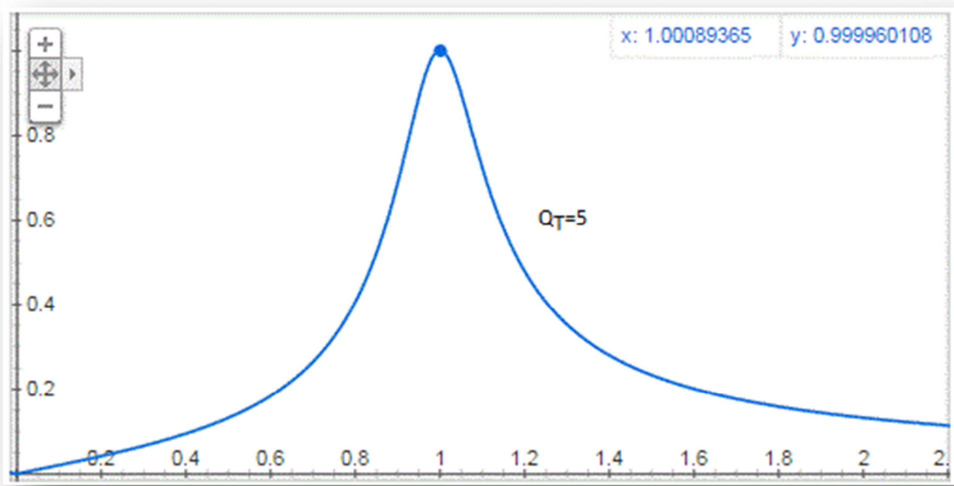


Fig.2 Impedance magnitude function $\frac{Z(\omega/\omega_0)}{R}$

The “half-power” or -3dB frequencies are those who satisfy the above equation. They are positive real quantities ω_A and ω_B. Solving for these we obtain:

$$\frac{\omega}{\omega_0} = \pm\frac{1}{2Q_T} \pm \sqrt{\frac{1}{4Q_T^2} + 1}$$

Hence:

$$\frac{\omega_B}{\omega_0} = \frac{1}{2Q_T} + \sqrt{\frac{1}{4Q_T^2} + 1}$$

and

$$\frac{\omega_A}{\omega_0} = -\frac{1}{2Q_T} + \sqrt{\frac{1}{4Q_T^2} + 1}$$

We observe that:

$$\frac{\omega_B - \omega_A}{\omega_0} = \frac{1}{Q_T}$$

This is the normalized -3dB bandwidth, and it yields an alternative expression for the Q factor of the tuned circuit:

$$Q_T = \frac{\omega_0}{\omega_B - \omega_A}$$

On the other hand, it can be easily verified that:

$$\frac{\omega_A \omega_B}{\omega_0^2} = 1$$

This is another important relationship involving ω_0, ω_B and ω_A, and is usually written in the form:

$$\omega_0 = \sqrt{\omega_A \omega_B}$$

It is interesting to notice that if $1/4Q_T^2 \ll 1$, or equivalently, $Q_T^2 \gg 1/4$, the resonant frequency ω_0 approaches the arithmetic mean of ω_A and ω_B, i.e.,

$$\omega_0 = \frac{\omega_A + \omega_B}{2}$$

There is still another relationship remaining to be calculated. It is useful when designing for frequency stability in an oscillator. It relates Q_T and the slope of the phase response $\phi(\omega)$ of the impedance function at the resonant frequency ω_0. Clearly, from the expression for $Z(j\omega)$:

$$\phi(\omega) = -tan^{-1}\left[Q_T \frac{\omega_0}{\omega}\left(\frac{\omega^2}{\omega_0^2} - 1\right)\right]$$

From tables for derivatives, on the other hand, we find that:

$$\frac{d}{dx} tan^{-1}(y) = \frac{1}{1 + y^2} \cdot \frac{dy}{dx}$$

Then:

$$\frac{d\phi(\omega)}{d\omega} = \frac{d}{d\omega} tan^{-1}\left[Q_T \frac{\omega_0}{\omega}\left(\frac{\omega^2}{\omega_0^2} - 1\right)\right]$$

$$= -\frac{Q_T\left[\frac{2}{\omega_0} - \frac{\omega_0}{\omega^2}\left(\frac{\omega^2}{\omega_0^2} - 1\right)\right]}{1 + {Q_T}^2 \frac{\omega_0^2}{\omega^2}\left(\frac{\omega^2}{\omega_0^2} - 1\right)^2}$$

If we evaluate the above expression at $\omega = \omega_0$ the following is obtained:

$$\phi'(\omega_0) = -\frac{2Q_T}{\omega_0}$$

or equivalently

$$Q_T = -\frac{1}{2}\omega_0 \phi'(\omega_0)$$

This is an interesting result, because the frequency stability of an oscillator depends to great extent on how quickly the phase response of the open-loop gain reacts to small frequency changes occurring in the system. In this sense, a large value for Q_T would be desirable for an oscillator employing a single tuned network of the type being studied herein. The reader should compare the normalized plots of $\phi(\omega)$ for $Q_T = 2$ and $Q_T = 5$ shown in Fig.3 in order to better appreciate how Q_T affects the shape of the phase response. Ordinates are in radians and abscissas in normalized frequencies.

The "Q" of a tuned circuit – A formal definition

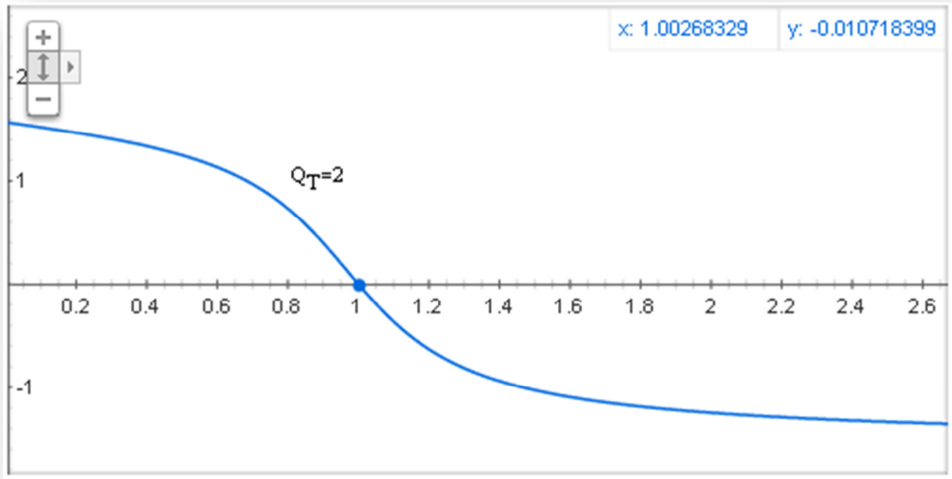
The formal definition for the quality factor, or Q factor, of a tuned circuit states that it is equal to 2π times the ratio of the peak energy stored in the tank to the energy dissipated per cycle by tank losses. It may be calculated for a series or parallel-tuned LC network and also for real-world reactances, which are lossy by nature. We will start calculating the Q factor of a coil, as defined above.

With reference to Fig.4, in the steady state, after the application of a sinusoidal current drive excitation to the circuit, the peak energy stored in the tank may be expressed by:

$$\varepsilon_s = \frac{1}{2}LI^2$$

The energy dissipated per cycle by coil losses is:

$$\varepsilon_{loss} = \frac{1}{2}I^2 rT$$



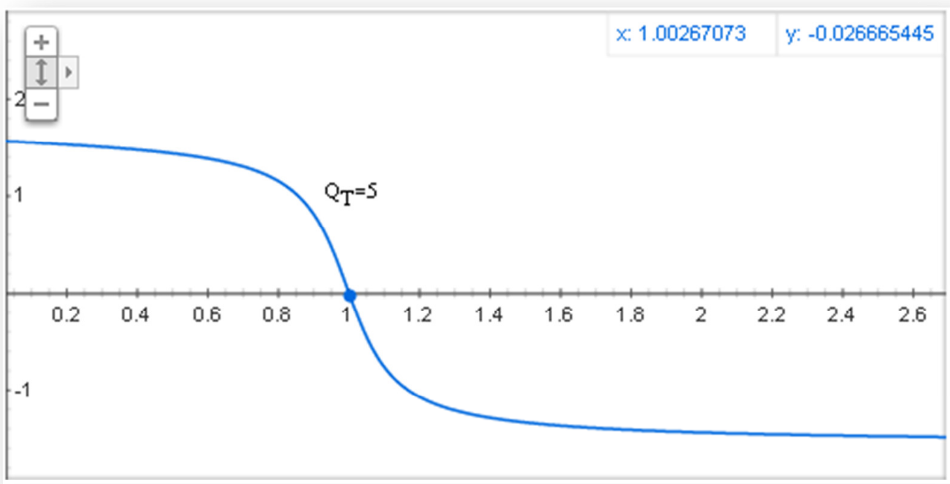


Fig.3 Plots of the normalized phase response $\phi(\omega/\omega_0)$ of the impedance function

where T is the duration of one cycle of the oscillatory current in seconds. Then:

$$Q_s = 2\pi \cdot \frac{\frac{1}{2}LI^2}{\frac{1}{2}I^2 rT} = \frac{2\pi f L}{r} = \frac{\omega L}{r}$$

being ω some radian frequency of interest.

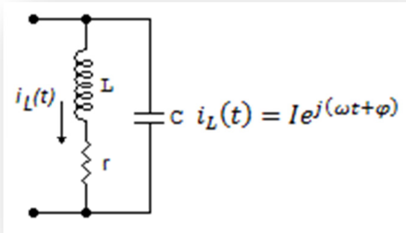


Fig.4 Losses in a coil as part of a tuned circuit

In the case of a parallel-tuned circuit (Fig.5), the Q is defined in a similar way:

$$Q_T = 2\pi \cdot \frac{\varepsilon_P}{\varepsilon_{loss}}$$

where ε_P is the peak energy stored by the capacitor and ε_{loss} is the energy dissipated per cycle by the tuned-circuit's losses. Then:

$$Q_T = 2\pi \cdot \frac{\frac{1}{2}CV^2}{\frac{1}{2}\frac{V^2}{R}T} = 2\pi fCR = \omega RC$$

being ω, again, some radian frequency of interest. If $\omega = \omega_0$, then Q_T is also given by:

$$Q_T = \frac{R}{\omega_0 L}$$

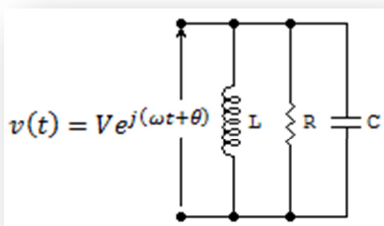


Fig.5 Losses in a parallel-tuned circuit

Now, we will try to derive a formula for the Q of a parallel-tuned circuit exhibiting losses in the capacitive and inductive branches. We shall refer to Fig.6.

The quality factor of a real-world inductor has been calculated to be:

$$Q_L = \frac{\omega L}{r_L}$$

The Q for the real-world capacitor may be calculated in a similar way as we did for the coil:

$$Q_C = 2\pi \cdot \frac{\varepsilon_P}{\varepsilon_{loss}}$$

where ε_P is the peak energy stored by the capacitor and ε_{loss} is the energy dissipated per cycle by capacitor losses. Here:

$$Q_C = 2\pi \cdot \frac{\frac{1}{2}CV_C^2}{\frac{1}{2}I_C^2 r_C T}$$

However, $V_C = I_C X_C$. Then,

$$Q_C = 2\pi \cdot \frac{\frac{1}{2}CI_C^2 X_C^2}{\frac{1}{2}I_C^2 r_C T}$$

$$= 2\pi \cdot \frac{Cf}{4\pi^2 f^2 C^2 r_C}$$

$$= \frac{1}{2\pi f C r_C}$$

$$= \frac{1}{\omega C r_C}$$

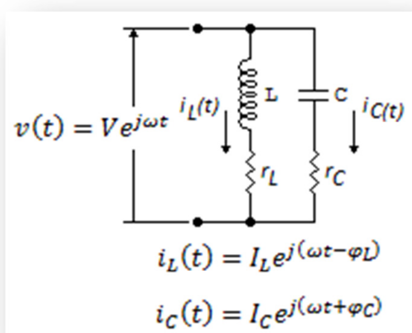


Fig.6 Losses in the inductive and capacitive branches

The effective Q may be defined as 2π times the ratio of the peak energy stored in the tank by C or L to the energy dissipated per cycle by r_L *and* r_C. We may write then:

$$Q_{eff} = 2\pi \cdot \frac{\frac{1}{2} L I_L^2}{\left(\frac{I_L^2 r_L}{2} + \frac{I_C^2 r_C}{2}\right) T}$$

Clearly, from Fig.6:

$$I_L^2 = \frac{V^2}{r_L^2 + \omega^2 L^2}$$

and

$$I_C^2 = \frac{V^2}{r_C^2 + \frac{1}{\omega^2 C^2}}$$

Hence:

$$Q_{eff} = 2\pi f \cdot \frac{\frac{L}{r_L^2 + \omega^2 L^2}}{\frac{r_L}{r_L^2 + \omega^2 L^2} + \frac{r_C}{r_C^2 + \frac{1}{\omega^2 C^2}}}$$

$$= \frac{\frac{\omega L}{r_L^2 + \omega^2 L^2}}{\frac{r_L}{r_L^2 + \omega^2 L^2} + \frac{r_C}{r_C^2 + \frac{1}{\omega^2 C^2}}}$$

$$\frac{1}{Q_{eff}} = \frac{1}{Q_L} + \frac{r_C}{r_C^2\left(1 + \frac{1}{\omega^2 r_C^2 C^2}\right)} \cdot \frac{r_L^2\left(1 + \frac{\omega^2 L^2}{r_L^2}\right)}{\omega L}$$

$$= \frac{1}{Q_L} + \frac{1}{r_C(1 + Q_C^2)} \cdot \omega L \left(\frac{1 + Q_L^2}{Q_L^2}\right)$$

At this point we need a result that will be proven later and that states that the impedance *Z(jω)* of the tuned network is real at the frequency $\omega = \omega_0$ for which:

$$\omega_0 L = \frac{1}{\omega_0 C} \cdot \frac{Q_C^2 + 1}{Q_C^2} \cdot \frac{Q_L^2}{Q_L^2 + 1}$$

Using this result, the Q_{eff} at that frequency can be found to be:

$$\frac{1}{Q_{eff}} = \frac{1}{Q_L} + \frac{1}{\omega_0 C r_C (1 + Q_C^2)} \cdot \frac{1 + Q_C^2}{Q_C^2}$$

$$= \frac{1}{Q_L} + \frac{Q_C}{1+Q_C^2} \cdot \frac{1+Q_C^2}{Q_C^2}$$

$$\frac{1}{Q_{eff}} = \frac{1}{Q_L} + \frac{1}{Q_C}$$

The Parallel-Tuned Circuit with Coil Losses

Now we return our attention towards the parallel-tuned circuit with power losses in the inductive branch (Fig.7).

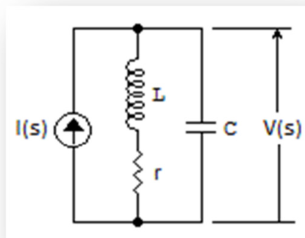


Fig.7 Parallel-tuned circuit with losses in the inductive branch

The admittance of the network is given by:

$$Y(s) = \frac{I(s)}{V(s)} = sC + \frac{1}{sL+r}$$

$$= \frac{s^2LC + srC + 1}{sL+r}$$

The impedance is, accordingly,

$$Z(s) = \frac{1}{Y(s)} = \frac{sL+r}{s^2LC+srC+1}$$

For $s = j\omega$:

$$Z(j\omega) = \frac{j\omega L + r}{(1-\omega^2LC) + j\omega Cr}$$

$$= \frac{(j\omega L + r)[(1-\omega^2LC) - j\omega Cr]}{(1-\omega^2LC)^2 + \omega^2C^2r^2}$$

$$= \frac{r + j\omega(L - \omega^2L^2C - Cr^2)}{(1-\omega^2LC)^2 + \omega^2C^2r^2}$$

The impedance *Z(jω)* is real at some frequency $\omega = \omega_0$ satisfying:

$$L - \omega_0^2 L^2 C - C r^2 = 0 \tag{1}$$

or equivalently:

$$\omega_0^2 LC = 1 - \frac{C}{L} r^2 \tag{2}$$

ω_0 is known as the phase resonant frequency, because as will be seen, the peak of the amplitude response occurs at some frequency *above* ω_0.

The Q factor of the coil is given by $Q_s = \omega_0 L / r$. Losses r can be estimated or measured, and so can be the values for Q_s in a real-world application. This said, Eq.(1) can be re-written as:

$$C(\omega_0^2 L^2 + r^2) - L = 0$$

or in the form:

$$C r^2 (Q_s^2 + 1) - L = 0$$

yielding:

$$\frac{Cr^2}{L} = \frac{1}{Q_s^2 + 1}$$

Substituting into Eq.(2) gives:

$$\omega_0^2 = \frac{1}{LC}\left(\frac{Q_s^2}{Q_s^2 + 1}\right)$$

$$= \frac{1}{\left(1 + \frac{1}{Q_s^2}\right) LC} \tag{3}$$

which suggests that an equivalent parallel resonance occurs between capacitor C and an inductor

$$L' = L\left(1 + \frac{1}{Q_s^2}\right) \tag{4}$$

the impedance at resonance being given by:

$$Z(j\omega_0) = \frac{L}{Cr} = R_P = (Q_s^2 + 1) r$$

From Eq.(3) we may conclude that $\omega_0 < 1/\sqrt{LC}$. Losses have shifted the resonance frequency below the value obtained when ideal reactances are considered in the tuned circuit.

At any other frequency the impedance of the network is given by:

$$Z(j\omega)=\frac{r+j\omega[L(1-\omega^2LC)-Cr^2]}{(1-\omega^2LC)^2+\omega^2C^2r^2}$$

$$=\frac{r+j\omega L[(1-\omega^2LC)-(1-\omega_0^2LC)]}{(1-\omega^2LC)^2+\omega^2LC(1-\omega_0^2LC)}$$

$$=\frac{r+j\omega L(\omega_0^2LC-\omega^2LC)}{(1-\omega^2LC)^2+\omega^2LC(1-\omega_0^2LC)}$$

With the aid of Eq.(3) we can write:

$$Z(j\omega)=\frac{r+j\omega L\left(\frac{Q_s^2}{Q_s^2+1}-\frac{\omega^2}{\omega_0^2}\cdot\frac{Q_s^2}{Q_s^2+1}\right)}{\left(1-\frac{\omega^2}{\omega_0^2}\cdot\frac{Q_s^2}{Q_s^2+1}\right)^2+\left(\frac{\omega^2}{\omega_0^2}\cdot\frac{Q_s^2}{Q_s^2+1}\cdot\frac{1}{Q_s^2+1}\right)}$$

From the definition for Q_s the following equivalence is certain:

$$\omega^2L^2=\frac{\omega^2}{\omega_0^2}Q_s^2r^2$$

Then, the magnitude of the impedance of the tuned-circuit or amplitude-response function can be written as:

$$|Z(j\omega)|=\frac{\sqrt{r^2+r^2\frac{\omega^2}{\omega_0^2}Q_s^2\left(\frac{Q_s^2}{Q_s^2+1}-\frac{\omega^2}{\omega_0^2}\cdot\frac{Q_s^2}{Q_s^2+1}\right)^2}}{\left(1-\frac{\omega^2}{\omega_0^2}\cdot\frac{Q_s^2}{Q_s^2+1}\right)^2+\left(\frac{\omega^2}{\omega_0^2}\cdot\frac{Q_s^2}{Q_s^2+1}\cdot\frac{1}{Q_s^2+1}\right)} \qquad (5)$$

Let:

$$\frac{Q_s^2}{Q_s^2+1}=1-\alpha$$

Then,

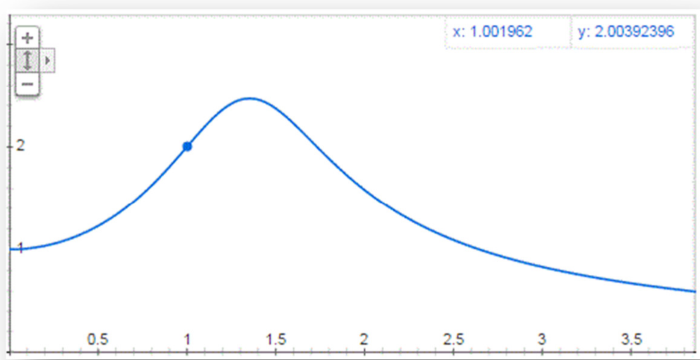
$$\alpha=\frac{1}{Q_s^2+1}$$

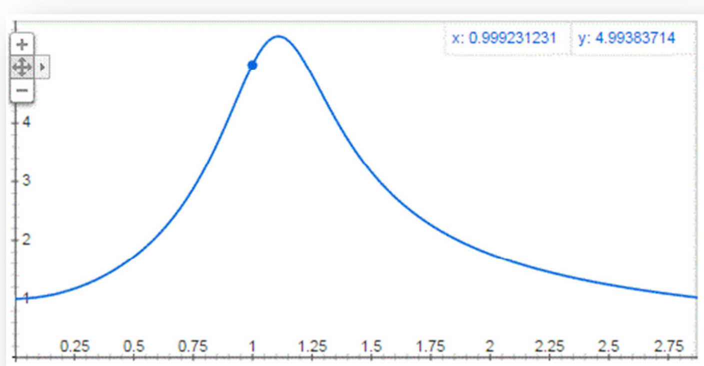
Substituting these definitions into Eq.(5) a more compact expression for $|Z(j\omega)|$ can be obtained that will facilitate the drawing of normalized graphs for the amplitude response. Accordingly:

$$|Z(j\omega)| = r\frac{\sqrt{1+\frac{\omega^2}{\omega_0^2}Q_s^2\left(\frac{Q_s^2}{Q_s^2+1}\right)^2\left(1-\frac{\omega^2}{\omega_0^2}\right)^2}}{\left(1-\frac{\omega^2}{\omega_0^2}\cdot\frac{Q_s^2}{Q_s^2+1}\right)^2+\left(\frac{\omega^2}{\omega_0^2}\cdot\frac{Q_s^2}{Q_s^2+1}\cdot\frac{1}{Q_s^2+1}\right)}$$

$$= r\frac{\sqrt{1+\frac{\omega^2}{\omega_0^2}Q_s^2(1-\alpha)^2\left(1-\frac{\omega^2}{\omega_0^2}\right)^2}}{\left[1-\frac{\omega^2}{\omega_0^2}(1-\alpha)\right]^2+\left[\frac{\omega^2}{\omega_0^2}(1-\alpha)\alpha\right]} \tag{6}$$

Normalized amplitude vs. frequency graphs for values of Q_s of 1, 2, 3, 5 and 10 are shown in Fig.8. Notice how the distance in normalized frequency units between the phase resonance frequency $(\omega/\omega_0 = 1)$ and that corresponding to the peak amplitude shortens as Q_s increases. Corresponding α values are 0.5, 0.2, 0.1, 0.0385 and 0.0099.





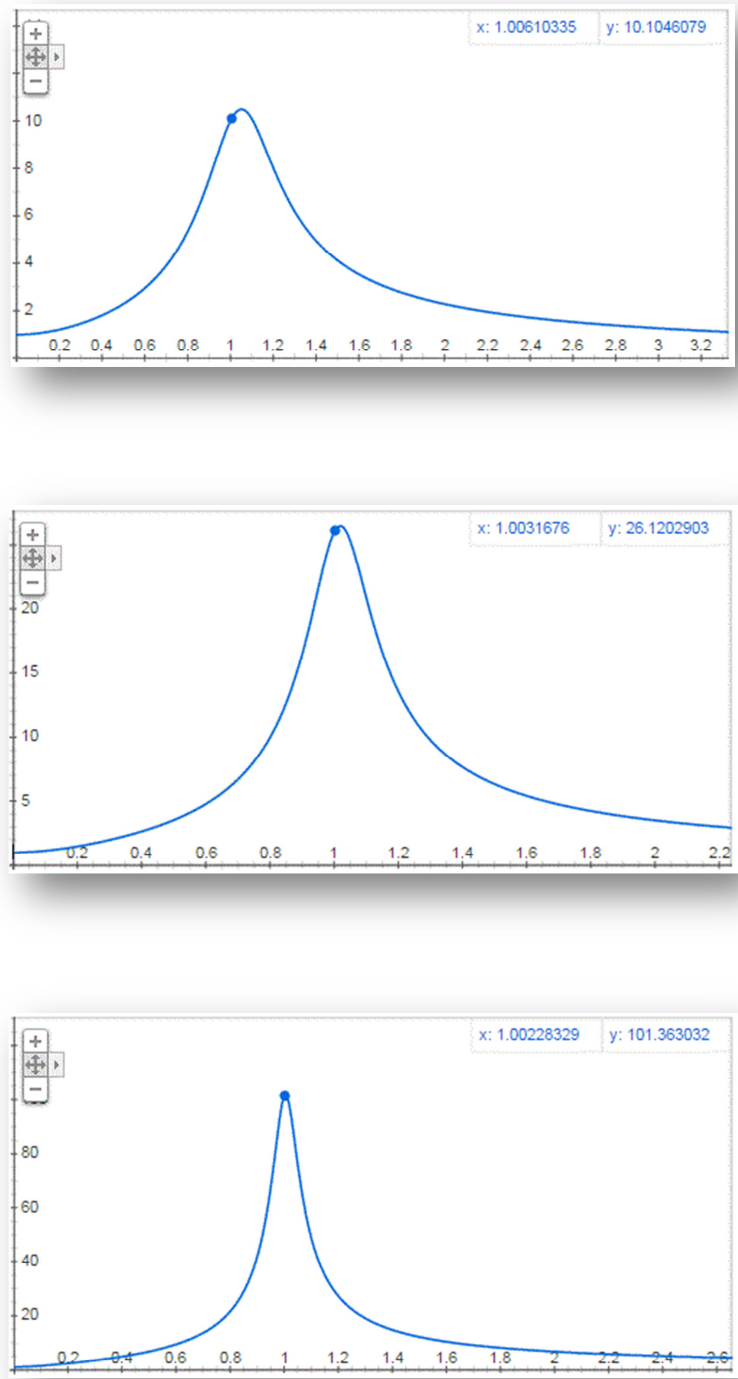


Fig.8 Normalized amplitude responses for Q_s values of 1, 2, 3, 5 and 10

The Parallel-Tuned Circuit with Lossy Reactances

The last part of our review-of-basics section will focus on the parallel-tuned network with losses in both branches. Fig.9 shows the circuit for analysis.

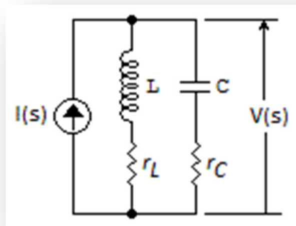


Fig.9 Parallel-tuned circuit with lossy reactances

As usual, we start calculating the admittance function:

$$\frac{I(s)}{V(s)} = \frac{1}{sL + r_L} + \frac{1}{\frac{1}{sC} + r_C}$$

$$= \frac{\left(\frac{1}{sC} + r_C\right) + (sL + r_L)}{(sL + r_L)\left(\frac{1}{sC} + r_C\right)}$$

$$= \frac{(1 + sCr_C) + sC(sL + r_L)}{(sL + r_L)(1 + sCr_C)}$$

$$= \frac{s^2LC + sC(r_L + r_C) + 1}{s^2LCr_C + s(L + Cr_Lr_C) + r_L}$$

The impedance is, accordingly,

$$Z(s) = \frac{V(s)}{I(s)} = \frac{s^2LCr_C + s(L + Cr_Lr_C) + r_L}{s^2LC + sC(r_L + r_C) + 1}$$

For $s = j\omega$:

$$Z(j\omega) = \frac{(r_L - \omega^2LCr_C) + j\omega(L + Cr_Lr_C)}{(1 - \omega^2LC) + j\omega C(r_L + r_C)}$$

$$Z(j\omega) = \frac{[(r_L - \omega^2LCr_C) + j\omega(L + Cr_Lr_C)][(1 - \omega^2LC) - j\omega C(r_L + r_C)]}{(1 - \omega^2LC)^2 + \omega^2C^2(r_L + r_C)^2}$$

$$Z(j\omega)=\frac{(r_L-\omega^2LCr_C)(1-\omega^2LC)+\omega^2C(r_L+r_C)(L+Cr_Lr_C)}{(1-\omega^2LC)^2+\omega^2C^2(r_L+r_C)^2}$$

$$+\frac{j\omega[(L+Cr_Lr_C)(1-\omega^2LC)-C(r_L+r_C)(r_L-\omega^2LCr_C)]}{(1-\omega^2LC)^2+\omega^2C^2(r_L+r_C)^2} \qquad (7)$$

Z(jω) is real when the imaginary part equates to zero, this is, when:

$$(L+Cr_Lr_C)(1-\omega_0^2LC)-C(r_L+r_C)(r_L-\omega_0^2LCr_C)=0$$

A little simplification leads to:

$$L-\omega_0^2L^2C=Cr_L^2-\omega_0^2LC^2r_C^2$$

We may write, following definitions given for the *Q*:

$$-\omega_0^2L^2C+\omega_0^2C^2r_C^2L=-L+Cr_L^2$$

$$-\omega_0^2L^2C+\frac{L}{Q_C^2}=-L+Cr_L^2 \qquad (8)$$

$$\frac{L}{Q_C^2}=-L+C(r_L^2+\omega_0^2L^2)$$

$$=-L+C(r_L^2+Q_L^2r_L^2)$$

$$=-L+Cr_L^2(1+Q_L^2)$$

$$\frac{1}{Q_C^2}+1=\frac{Cr_L^2}{L}(1+Q_L^2)$$

Then:

$$\frac{Cr_L^2}{L}=\frac{1}{Q_L^2+1}\cdot\frac{Q_C^2+1}{Q_C^2} \qquad (9)$$

Dividing Eq.(8) by *L* and substituting Eq.(9) in the resultant expression yields, first:

$$-\omega_0^2LC+\frac{1}{Q_C^2}=-1+\frac{Cr_L^2}{L}$$

and then:

$$\omega_0^2LC=1+\frac{1}{Q_C^2}-\frac{1}{Q_L^2+1}\cdot\frac{Q_C^2+1}{Q_C^2} \qquad (10)$$

The last expression can be written as:

$$\omega_0^2 LC = \frac{Q_L^2}{Q_L^2+1} \cdot \frac{Q_C^2+1}{Q_C^2} \tag{11}$$

which gives explicitly the frequency at which the impedance $Z(j\omega)$ is a real quantity, if Q_L and Q_C are known figures.

Equation (11) can be put in the equivalent form:

$$\omega_0^2 = \frac{1}{L\left(1+\frac{1}{Q_L^2}\right) \cdot C\left(\frac{Q_C^2}{Q_C^2+1}\right)} \tag{12}$$

which suggests that an equivalent parallel resonance occurs between an inductor

$$L' = L\left(1+\frac{1}{Q_L^2}\right)$$

and a capacitor

$$C' = C\left(\frac{Q_C^2}{Q_C^2+1}\right)$$

Furthermore, it can be easily shown that:

$$\omega_0^2 LC \cong 1 + \frac{1}{Q_C^2} - \frac{1}{Q_L^2} \tag{13}$$

is a good approximation to Eq.(10) when Q_C and Q_L are greater than 3. This result has been used by radio experimenters in the past to propose simple tuning controls using variable resistors in series with one of the tuned-circuit branches, usually the capacitive branch. While this helps when tuning to a precise frequency, it contributes to more RF power losses in the circuit, an issue that can be easily corrected with the use of positive feedback or regeneration. It must be born in mind that regeneration amplifies the effective Q values of the circuit to rather large numbers, so the tuned frequency tends to the ideal value $\omega_0^2 = 1/LC$, or equivalently, $\omega_0 = 1/\sqrt{LC}$.

It can be shown that for frequencies in the neighborhood of ω_0, series loss resistors r_L and r_C in Fig.9 can be transformed into larger lossy resistors in parallel with ideal L and C reactances. We would like to calculate the impedance of the tuned circuit at frequency ω_0.

From Eq.(7) and for the case where $Q_L = Q_C$ it is interesting to notice that:

$$Q_L = \frac{\omega_0 L}{r_L} = \frac{1}{\omega_0 C r_C} = Q_C$$

condition that forces expression (11) to be unity. Likewise, series loss resistors must be identical, or $r_L = r_C$. This being the case, Eq.(7) reduces to:

$$Z(j\omega_0) = \frac{\omega^2 C(r_L + r_C)(L + Cr_L r_C)}{\omega^2 C^2 (r_L + r_C)^2}$$

or

$$Z(j\omega_0) = \frac{L + Cr_L r_C}{C(r_L + r_C)}$$

$$= \frac{L\left(1 + \frac{1}{Q^2}\right)}{2Cr}$$

$$= \frac{\omega_0 L\left(1 + \frac{1}{Q^2}\right)}{2\omega_0 Cr}$$

$$= \frac{Qr\left(1 + \frac{1}{Q^2}\right)}{2} \cdot Q$$

$$= \frac{r(Q^2 + 1)}{2}$$

where $r = r_L = r_C$ and $Q = Q_L = Q_C$. The transformed loss resistors are each equal to:

$$R_P = r(Q^2 + 1)$$

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